LAND AND ENGINEERING SURVEYING

For Students and Practical Use

ORIGINALLY WRITTEN

BY

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REVISED BY

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PREFACE

Mr. F. E. Dixon, in his preface to the last edition of this work, said, "The 'Rudimentary Treatise' of the late Mr. Baker on Land and Engineering Surveying" has been so long and so widely known as an accepted text-book in schools and colleges, and has been so generally recommended by professional authorities, that it is needless to dwell upon its merits. But the conditions under which the training of Students in Surveying, as well as the work of the Surveyor, is now carried on, are so different from those prevailing when the book was first issued, that a revision of the volume was felt to be necessary, and by arrangement with the Publishers, I have undertaken the preparation of this edition."

By the efflux of time the same necessity has again arisen, and Mr. Dixon's remarks are equally applicable. Mr. Dixon also says that the volume "being recommended by the Councils of the Institution of Surveyors, the Royal Agricultural Society of England, and the Incorporated Association of Municipal and County Engineers, examples of the Examination Papers issued by them have been introduced; and I may say that in re-writing the work I have endeavoured so to adapt and extend the contents of the volume to meet the requirements of those bodies." In the present edition these features have all been retained. In any corrections or additions that I have made the requirements of any

particular body in their examinations have not been considered, as this had already been provided for. I have therefore contented myself with the requirements of the student in his search for knowledge.

I have made several corrections throughout the text, but chiefly with regard to the adjustment of instruments. The methods I have given are thoroughly reliable and apply to the most modern types of theodolites and levels as well as to the older patterns. As this subject is important to the student in examinations, and more important still in his practice, I have endeavoured to present it in such a manner that it can be both understood and applied. This cannot be said of the descriptions to be found in most text-books, and if I have succeeded in my endeavour the student will be grateful.

The reference to latitude and longitude has been rewritten. This is of necessity very brief, but it is hoped that what has been said will at least be understandable.

I might add that I have cut out references to triangulation when chain surveys are meant. To use the terms usually associated in the mind of a real surveyor with triangulation, when chain surveys are intended, is, in my opinion, very confusing to the student. In my experience as an examiner in Surveying I have known cases where this looseness in terms has led candidates into the most ridiculous positions, through no fault of their own.

To again quote Mr. Dixon, "it is hoped that the work may be found sufficiently brought 'up to date' to prove a satisfactory progressive Introduction to the Practice and Principles of Surveying, and be found increasingly useful by those for whose assistance it is designed."

GEO. LIONEL LESTON.

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LAND AND ENGINEERING SURVEYING.

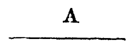
CHAPTER I.

PRACTICAL GEOMETRY.

Definitions.

1. *A point* has no dimensions, neither length, breadth, nor thickness.

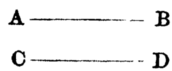
2. *A line* has length only, as A. A line may be straight or curved.



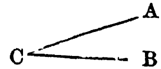
3. *A surface* or *plane* has length and breadth, as B. A surface may be either flat or curved.

4. *A straight line* lies evenly between its extreme points, as A B.

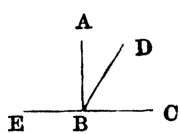
5. *Parallel lines* are such as are in the same plane, and never meet when prolonged, as A B and C D.



6. *An angle* is formed by the meeting of two lines not being in the same straight line, as A C, C B. It is called the angle A C B.



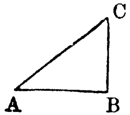
7. *A right angle* is formed by a straight line standing on another straight line, and making the adjacent angles equal to one another, as the angles A B E and A B C, which will be right angles. The line A B will then be perpendicular to E C.



8. *An acute angle* is less than a right angle, as **D B C.**

9. *An obtuse angle* is greater than a right angle, as D B E.

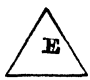
10. *A triangle* is a figure included by three straight lines.



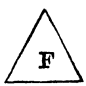
11. *A right-angled triangle* is one which has a right angle, as A B C. The side A C, opposite the right angle, is called the *hypotenuse*; the sides A B and B C are respectively called the base and perpendicular.

12. *An obtuse-angled triangle* is that which has an obtuse angle, as the angle at B.

13. *An acute-angled triangle* is that which has all its three angles acute, as D.

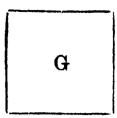


14. *An equilateral triangle* is that which has three equal sides, as E.



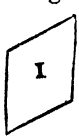
15. *An isosceles triangle* is that which has two equal sides, as F.

16. *A quadrilateral figure* is a figure bounded by four straight lines; when its opposite sides are parallel and equal, it is called a *parallelogram.*



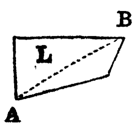
17. *A square* has all its sides equal, and all its angles right angles, as G.

18. *A rectangle* is a parallelogram with all its angles right angles, as B (see figure to Definition 3) or G.



19. *A rhombus* is a parallelogram having all its sides equal, but its angles are not right angles, as I.

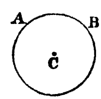
20. *A rhomboid* is a parallelogram having its opposite sides equal, but all its sides are not equal, nor its angles right angles, as K.



21. *A trapezium* is a figure bounded by four straight lines, no two of which are parallel to each other, as L. A line connecting any two of its opposite angles is called the *diagonal*, as A B.

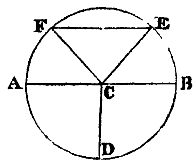
22. *A trapezoid* is a quadrilateral, having two of its opposite sides parallel, and the remaining two not, as M.

23. *Polygons* are figures having more than four sides, and receive particular names, according to the number of their sides. Thus, a *pentagon* has five sides; a *hexagon*, six; a *heptagon*, seven; an *octagon*, eight; &c. They are called *regular* polygons when all their sides and angles are equal; when otherwise, *irregular* polygons.



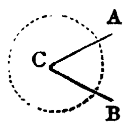
24. *A circle* is a plane figure, bounded by a curved line, called the *circumference*, which is everywhere equidistant from a point C within, called the *centre*.

25. *An arc of a circle* is any part of the circumference, as F E.



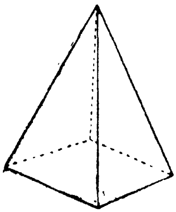
26. *The diameter of a circle* is a straight line A B, passing through the centre C. It divides the circle into two equal parts, each of which is called a *semicircle*. Half the diameter, as A C or C B, is called the *radius*. If a radius C D be drawn at right angles to A B, it will divide the semicircle into two equal parts, each of which is called a *quadrant*, or one-fourth of a circle. A *chord* is a straight line joining the extremities of an arc, as F E. It divides the circle into two unequal parts called *segments*. If the radii C F, C E be drawn, the space bounded by these radii and the arc F E will be the *sector of a circle.*

27. The circumference of every circle is divided into 360 equal parts called degrees, and each degree into 60 minutes, each minute into 60 seconds, &c. Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.

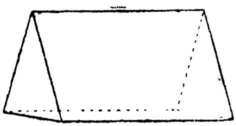


28. *The measure of an angle* is an arc of any circle contained between the two lines which form the angle, the angular point being the centre; and it is estimated by the number of degrees contained in that arc. Thus the arc A B, the centre of which is C, is the measure of the angle A C B. If the angle A C B contain 42 degrees, 29 minutes, and 48 seconds, it is thus written: $42° \; 29' \; 48''$

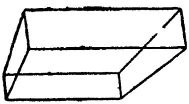
29. *A solid* is that which has length, breadth, and thickness. That which bounds a solid is *a superficies*.



30. *A pyramid* is a solid figure, contained by three or more triangles which meet at a point, and by another rectilinear figure.

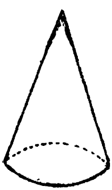


31. *A prism* is a solid figure contained by plane figures, of which two that are opposite are equal, similar, and parallel to one another; and the others are parallelograms.

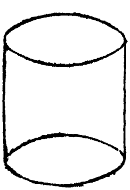


32. *A prismoid* is a solid having for its ends any two parallel plane rectilineal figures of the same number of sides, and having all its faces trapezoids.

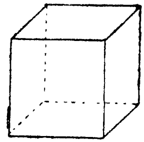
33. *A sphere* is a solid figure, described by the revolution of a semicircle about its diameter, which remains fixed.



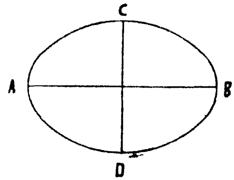
34. *A cone* is a solid figure, described by the revolution of a right-angled triangle about one of the sides containing the right angle, which side remains fixed.



35. *A cylinder* is a solid figure, described by the revolution of a right-angled parallelogram about one of its sides, which remains fixed.



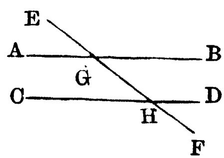
36. *A cube* is a solid figure contained by six equal squares.



37. *An ellipse* is the curve traced out by a point, which moves in such a manner that its distance from a given point, called the *focus*, is in a constant ratio of less inequality to its distance from a given straight line, called the *directrix*. The diameter A B is called the *transverse diameter* ; the diameter C D the *conjugate diameter.*

Theorems.

I. *On the same base, and on the same side of it, there cannot be two triangles having their sides which are terminated at one extremity of the base equal to one another, and likewise those which are terminated at the other extremity equal to one another.* (Euc. I. 7.)



II. *If two straight lines cut one another, the vertical,* or opposite, *angles are equal.* (Euc. I. 15.) *Thus the angle* A G E = *angle* H G B, *and* E G B = A G H.

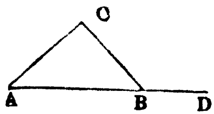
III. (*See last figure*).—*A straight line* E F, *cutting two parallel straight lines* A B, C D, *makes the alternate angles equal, &c. :—thus the angles* A G H, G H D *are equal ; also the exterior angle* E G B *is equal to the interior and opposite angle* G H D *; also the angles* B G H *and* G H D *are together equal to two right angles.* (Euc. I. 29.)

IV. *The greatest side of every triangle is opposite the greatest angle.* (Euc. I. 18.)

V. *If two sides of one triangle are equal to two sides of another triangle, each to each, and the angle contained by the two*

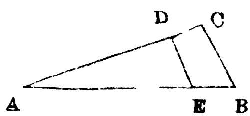
sides of the one equal to the angle contained by the two sides of the other, the triangles will be equal in all respects. (Euc. I. 4.)

VI. *If two triangles have two angles of the one equal to two angles of the other, each to each, and one side equal to one side, namely, either the sides adjacent to the equal angles, or sides which are opposite to equal angles in each, then shall the other sides be equal, each to each, and also the third angle of the one equal to the third angle of the other.* (Euc. I. 26.)



VII. *Let the side* A B *of the triangle* A B C *be produced to* D, *the exterior angle* C B D *is equal to the interior angles at* A *and* C ; *also the three interior angles of the triangle are equal to two right angles.* (Euc. I. 32.) *Whence any two angles of a triangle being given the third becomes known.*

VIII. *Let* A B C *be a right-angled triangle* (see figure to Definition 11, p. 2), *having a right angle at* B ; *then, the square on the side* A C *is equal to the sum of the squares on the sides* A B, B C. (Euc. I. 47.) *Whence any two sides of a right-angled triangle being given the third becomes known.*



IX. *In any triangle* A B C, *let* D E *be drawn parallel to one of its sides,* C B ; *then,* A B *is to* A E *as* B C *is to* D E ; *and the triangles* A D E *and* A C B *are said to be similar.* (Euc. VI. 2.)

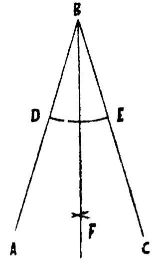
X. *Let* A B C, A E D (see last figure) *be similar triangles, then, the triangle* A B C *is to the triangle* A E D *as the square of* A B *is to the square of* A E ; *that is, similar triangles are to one another in the duplicate ratio of their homologous, or like, sides.* (Euc. VI. 19.)

XI. *All similar figures are to one another in the duplicate ratio of their homologous, or like, sides.* (Euc. VI. 20.)

XII *All similar solids are to one another as the cubes of their like linear dimensions.*

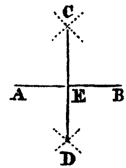
Problems.

I. *To divide a given angle* A B C *into two equal parts.*



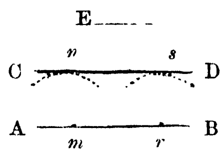
From the point B with any radius describe the arc D E. From D and E as centres describe arcs of equal radius cutting each other in F. Join B F, which will bisect the angle required.

II. *To divide a given straight line* A B *into two equal parts.*



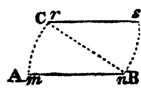
From the centres A and B, with any radius greater than half A B, describe arcs, cutting each other in C and D; draw C D, and it will cut A B in the middle point E.

III. *At a given distance* E, *to draw a straight line* C D *parallel to a given straight line* A B.



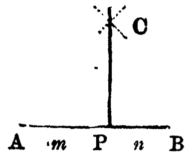
From any two points *m* and *r*, in the line A B, with a radius equal to E, describe the arcs *n* and *s* :—draw C D to touch these arcs, without cutting them, and it will be parallel to A B.

IV. *Through a given point r, to draw a straight line* C D *parallel to a given straight line* A B.



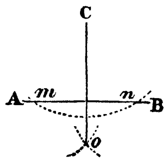
From any point *n* in the line A B, with the distance *n r*, describe the arc *r m*, draw the chord *m r* :—from centre *r*, with the same radius, describe the arc *n s* :—draw the chord *n s* equal to the chord *m r* :—through *r* and *s* draw C D, which is the parallel required.

V. *From a given point* P, *in a straight line* A B, *to erect a perpendicular.*



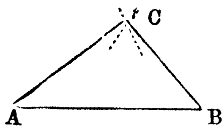
On each side of the point P take any two equal distances, P *m*, P *n*, on the line A B (prolonged if necessary); from the points *m* and *n*, as centres, with any radius greater than P *m*, describe two arcs cutting each other in C; through C, draw C P, and it will be the perpendicular required.

VI. *From a given point* C *to let fall a perpendicular to a given line.*



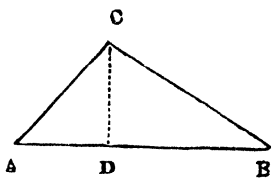
From C, as a centre, describe an arc to cut A B (prolonged if necessary) in *m* and *n*;—with centres *m* and *n*, and the same or any other radius, describe arcs intersecting in *o*: through C and *o* draw C *o*, the perpendicular required.

VII. *To construct a triangle with three given straight lines.* (Euc. I. 22.)



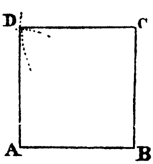
Let the three given lines be 5, 4 and 3 yards. From any scale of equal parts lay off the base A B = 5 yards; with the centre A and radius A C = 4 yards, describe an arc; with centre B and radius C B = 3 yards, describe another arc cutting the former arc in C:—draw A C and C B; then A B C is the triangle required.

VIII. *Given the base and perpendicular, with the place of the latter on the base, to construct the triangle.*



Let the base A B = 7, the perpendicular C D = 3, and the distance A D = 2. Make A B = 7 and A D = 2;—at D erect the perpendicular D C, which make = 3: —draw A C and C B; then A B C is the triangle required.

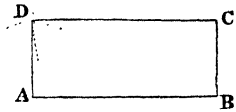
IX. *To describe a square, whose side shall be of a given length.*



Let the given line A B be 3 chains. At the end B of the given line erect the perpendicular B C (by Prob. V.), which make = A B:—with A and C as centres, and radius A B, describe arcs cutting each other in D: draw A D, D C and the square will be completed.

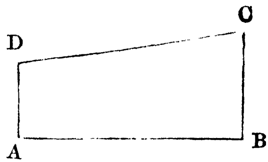
X. *To describe a rectangle having a given length and breadth.*

Let the length A B = 5 chains, and the breadth B C = 2. At B erect the perpendicular B C and make it = 2 :—from A as centre, with radius B C, describe an arc; and from C as centre, with radius A B, describe another arc, cutting the former in D :—join A D, D C to complete the rectangle.



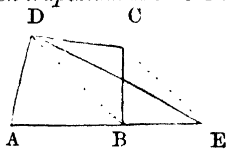
XI *The base and two perpendiculars being given, to construct a trapezoid.*

Let the base A B = 6, and the perpendiculars A D and B C, 2 and 3 chains respectively. Draw the perpendiculars A D, B C, as given above, and join D C, thus completing the trapezoid.



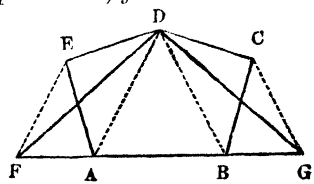
XII. *To make a triangle equal to a given trapezium* A B C D.

Draw the diagonal D B, and C E parallel to it, meeting A B prolonged in E :—join D E ; then shall the triangle A D E be equal to the trapezium A B C D.



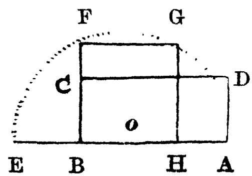
XIII. *To make a triangle equal to the figure* A B C D E A.

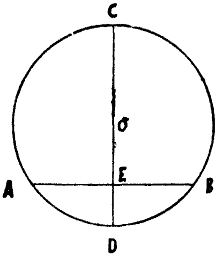
Draw the diagonals D A, D B, and the lines E F, C G, parallel to them, meeting the base A B produced both ways, in F and G :—join D F, D G : then the triangle D F G will be equal to the given figure A B C D E A.



XIV. *To make a square equal to a given rectangle* A B C D.

Produce one side A B till B E be equal to B C :—bisect A E in *o* ; from which as a centre, with radius A *o*, describe a semicircle, and prolong B C to meet it in F :—on B F describe the square B F G H, and it will be equal to the rectangle A B C D, as required.





XV. *To find the centre of a given circle.*

Draw any chord A B and bisect it at right angles by the straight line C D, which will be a diameter. Bisect C D in O, which will be the centre of the circle.

XVI. *To construct an ellipse.*

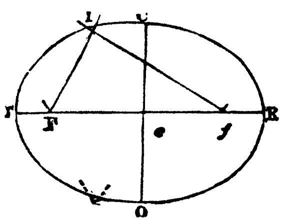
RULE I.—*When one of the diameters is given.* Divide the given area successively by ·7854 and the given diameter, and the quotient will be the other diameter.

RULE II.—*When the two diameters are required to be in a given proportion or ratio.* Divide the given area successively by ·7854 and the product of the terms of the ratio, and the square root of the quotient, multiplied separately by the terms of the ratio will give the two diameters.

Example 1.—Construct an ellipse containing 4 acres, and with a transverse diameter T R of 9 chains.

By Rule I. $$\frac{400000}{\cdot 7854 \times 900} = 566 \begin{cases}\text{links} = \text{C O, the} \\ \text{conjugate diameter.}\end{cases}$$

Lay off from the centre *c* of the ellipse, on the transverse diameter T R, the distances *c* F, *c f*, each equal to the square root of the difference of the squares of the two semi-diameters; take a strong flexible cord, equal to T R, and fix its ends at F, *f*; extend the cord to any point, I. so that it may take the position F I *f*, and keeping the cord continually stretched, trace the elliptical curve T I C R O, which will be the required boundary.



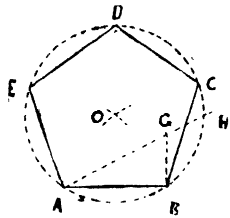
In this case $c\,F = cf = \sqrt{450^2 - 283^2} = 350$ links.

Example 2.—Lay out an ellipse to contain 2*a*. 0*r*. 32*p*., the proportion of the diameters of which shall be as 7 to 4.

By Rule II. $\sqrt{\frac{220000}{4 \times 7 \times \cdot 7854}} = 100$ links; and $100 \times 7 = 700$ links, the transverse diameter, and $100 \times 4 = 400$ links, the conjugate.

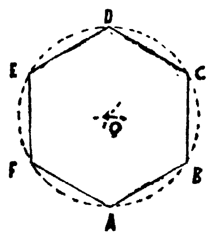
XVII. *Upon a given line A B to construct a regular pentagon.*

From the point B draw B G perpendicular to A B and equal to half A B. Join A G and produce it to H, making G H = G B.



From the points A and B with the radius B H describe arcs intersecting in O, which will be the centre of the circle circumscribing the required pentagon. With centre O and the same radius describe the circumscribing circle A B C D E. Set off the lines B C, C D, D E, and E A, all equal to A B, and intersecting the circumference in the points C, D, E. Then the figure A E D C B will be the pentagon required.

XVIII. *Upon a given line A B to construct a regular hexagon.*

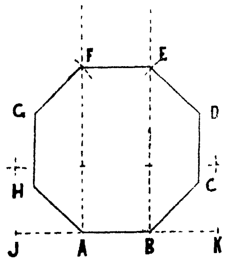


From the points A and B as centres, with the radius A B, describe arcs intersecting in O.

With the centre O and the same radius describe the circle A B C D E F, which will be the circle circumscribing the required hexagon. Set off the lines B C, C D, D E, E F, and F A, all equal to A B, and intersecting the circumference in the points C D E F. Then the figure A B C D E F will be the hexagon required.

XIX. *Upon a given line A B to construct a regular octagon.*

From the points A and B draw A F and B E perpendicular to A B. Produce A B to J and K.



Bisect the angles F A J and E B K by the lines A H and B C, each equal to A B.

Draw G H and C D parallel to A F, making them equal to A B. Draw G F parallel to B C, and E D parallel to A H, making them equal to A B; join E F. Then A B C D E F G H will be the octagon required.

CHAPTER II.

MENSURATION.

Uniformity in Measures commanded by Statute.—Uniformity in measures has been from time to time commanded by Statute, but until the present century without the contemplated results, "local" measures, varying in different parts of the country, continuing in force. But by 5 Geo. IV. c. 74, it was enacted that on and after the first of May, 1825, the standard yard was to be the unit of measurement, and all measurements, lineal and superficial, were to be multiples and sub-multiples of it, the acre to contain 160 perches or 4,840 square yards.

Since that date these measures have gradually superseded the local measures. The principal local measures are, however, sometimes met with, and some measures of the acre are for this reason set down below.

Table of Linear Measure.

Inches.	Links.	Feet.	Yards.	Poles.	Chains.	Furlongs.	Mile.
7·92	1						
12	1·515	1					
36	4·545	3	1				
198	25	16½	5½	1			
792	100	66	22	4	1		
7,920	1,000	660	220	40	10	1	
63,360	8,000	5,280	1,760	320	80	8	1

Table of Superficial Measure.

Sq. Inches.	Sq. Links.	Sq. Feet.	Sq. Yds.	Sq. Perches.	Sq. Chs.	Roods.	Acres.	Sq. Mile.
144	2·296	1						
1,296	20·661	9	1					
39,204	625	272¼	30¼	1				
627,264	10,000	4,356	484	16	1			
1,568,160	25,000	10,890	1,210	40	2½	1		
6,272,640	100,000	43,560	4,840	160	10	4	1	
—	64,000,000	27,878,400	3,097,600	102,400	6,400	2,560	640	1

Table of Solid Measure.

Cubic Inches.	Cubic Feet.	Cubic Yard.
1,728	1	
46,656	27	1

Local Measures of the Acre.

		Sq. Yds.	Statute Acre.
1 Lancashire acre*	contains 160 perches of 49 sq. yds.	= 7,840	= 1·620
1 Irish ,,	,, 160 ,, 49 ,,	= 7,840	= 1·620
1 Cheshire ,,	,, 160 ,, 64 ,,	= 10,240	= 2·116
1 Scotch ,,	,, 10 sq. chains of 74 feet	= $6{,}084\frac{4}{9}$	= 1·257

Table of Hay and Straw Measure.

1 truss of straw = 36 lbs.
1 truss of old hay = 56 lbs.
1 truss of new hay to Sept. 1st. = 60 lbs.
1 load of straw = 36 trusses = 11 cwt. 2 qr. 8 lbs.
1 load of old hay = 36 trusses = 18 cwt. 0 qr. 0 lbs.
1 load of new hay = 36 trusses = 19 cwt. 1 qr. 4 lbs.

Table of Dry or Corn Measure.

Wheat and other cereals are commonly sold by weight, the bushel being thus reckoned :—

1 bushel wheat (English) = 63 lbs. (Foreign) = 62 lbs.
1 ,, barley ,, = 52 to 56 lbs. (French) = 52½ lbs.
1 ,, oats ,, = 40 to 42 lbs. (Foreign) = 38 to 40 lbs.
1 ,, rye or maize = 60 lbs.

NOTE.—Grain of all kinds is frequently sold by the stone.

* The Lancashire and Irish acres are commonly spoken of as equal to 1·619 statute acres, the actual measurement to five places of decimals being 1·61983.

Table of Timber or Wood Measure.

40 cubic feet rough, or 50 cubic feet squared	= 1 load.
50 feet planks	= 1 load.
100 superficial feet	= 1 square of flooring.

Width of battens 7 inches; of deals, 9 inches; planks, 2 to 4 inches, and 10 or 11 inches wide.

Hydraulic Memoranda.

1 cubic foot of water	= 6·23 gallons.
1 gallon ,,	= 10 lbs. avoirdupois = 70,000 grains Troy.
1 inch of rainfall	= 22,622 gallons per acre.

Velocity and Discharge of Water in Pipes and Open Channels.

V = velocity in feet per second.
D = diameter of pipe in feet.
A = area of channel.

$$R = \text{hydraulic mean depth} = \frac{\text{area of cross section}}{\text{wetted perimeter.}} = \text{in pipes } \frac{D}{4}$$

$$S = \text{sine of inclination or } \frac{\text{total fall}}{\text{total length.}}$$

Q = discharge in gallons per minute.

(i) *Velocity and Discharge in Pipes.*

$$V = 140\sqrt{RS} - 11\sqrt[3]{RS}$$
$$Q = V \times 293{\cdot}7286\ D^2$$

(ii) *Velocity and Discharge in Open Channels.*

$$V = 93\sqrt{RS}$$
$$Q = AV$$

Pressure in Pipes.

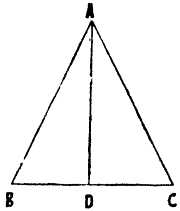
H = head of water in feet.
P = pressure in lbs. per square inch.

$$P = 0{\cdot}433\ H.$$

Values of French Measures.

1 mètre	= 39·37 inches.
1 kilomètre	= 1093·62 yards.
1 hectare	= 2 471 acres.

Lengths, Areas, and Solid Contents.



I. *The Triangle.*

RULE I.—Area = base multiplied by one-half the perpendicular; thus area of $ABC = BC \times \frac{AD}{2}$

RULE II. (when the three sides of the triangle are given).—From half the sum of the three sides subtract each side separately; multiply the half-sum and the three remainders together: the square root of the product will be the area.

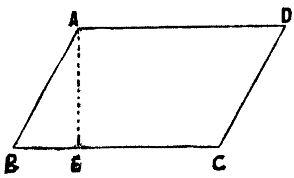
The area is always, where possible, computed by the first rule.

II. *The Square.*

RULE.—Area = product of any side multiplied by itself.

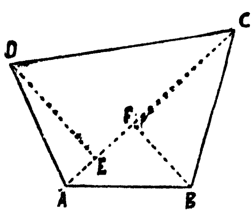
III. *The Rectangle.*

RULE.—Area = length multiplied by breadth.



IV. *The Parallelogram.*

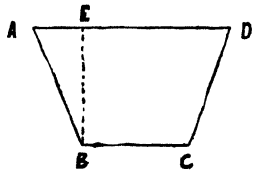
RULE.—Area = base multiplied by perpendicular height; thus area of parallelogram $ABCD = BC \times AE$.



V. *The Trapezium.*

RULE. — Divide the trapezium into two triangles, always taking the longest diagonal, and draw perpendiculars to this diagonal from the apex of each so-formed triangle. Then area = half the product of the sum of the two perpendiculars multiplied by the diagonal; thus, area of trapezium $ABCD = AC \times \frac{(BF + DE)}{2}$.

VI. *The Trapezoid.*



RULE.—Area = one-half of the sum of the two parallel sides multiplied by the perpendicular distance between them; thus area of trapezoid A B C D $= \frac{AD + BC}{2} \times BE.$

VII. *Regular Polygons.*

RULE.— Area = half the number of sides multiplied by length of one side multiplied by radius of inscribed circle.

A Table of Regular Polygons,

with their names, areas, and radii of their circumscribing and inscribed circles, the sides of the polygons being unity.

No. of Sides.	Names.	Areas.	Radius of Circumscribing Circle.	Radius of Inscribed Circle.
3	Triangle (equilateral) .	0·433	0·577	0·289
4	Square	1·	0·707	0·5
5	Pentagon	1·72	0·851	0·688
6	Hexagon	2·598	1·	0·866
7	Heptagon	3·634	1·152	1·038
8	Octagon	4·828	1·306	1·207
9	Nonagon	6·182	1·462	1·374
10	Decagon	7·694	1·618	1·539
11	Undecagon	9·366	1·775	1·703
12	Duodecagon	11·196	1·932	1·866

Given the side, to find the area from the Table.

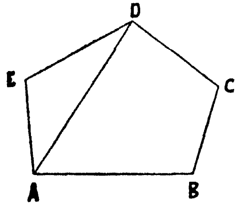
RULE.—Square the side of the polygon and multiply it by the corresponding area in the Table.

Given the area, to find the length of the side and the radius of the circumscribing circle.

RULE.—Divide the area of the proposed polygon by its corresponding area in the Table, and the square root of the result will be the length of its sides. Multiply the side just found by the corresponding radius in the column marked "Radius of Circumscribing Circle," and the result

will be the radius of the circle that circumscribes the required polygon.

VIII. *Irregular Polygons.*



RULE.—Divide the polygon into the most convenient trapeziums and triangles; find the area of each, and the sum of these areas will be the area of the polygon.

Thus area of A B C D E = area of triangle A E D + area of trapezium D A B C.

IX. *The Circle.*

RULE.—Circumference = diameter of circle multiplied by 3·1416.

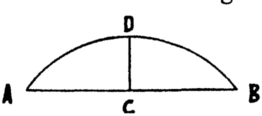
RULE.—Length of any arc = (nearly) one-third of difference between eight times chord of half the arc and the chord of the whole arc.

NOTE.—Half the chord of the whole arc, the chord of half the arc, and the versed sine (or height), are sides of a right-angled triangle; so that by Theorem viii., p. 6, any two of these being given the third can be found.

RULE.—Area of circle = diameter squared × ·7854.

RULE.—Area of sector of circle = half the product of length of arc multiplied by radius of sector. (See illustration to Definition No. 26, p. 3, where area of sector C E F = length of arc F E multiplied by $\frac{CF}{2}$.)

RULE.—Area of segment of circle = two-thirds of product of chord and height of segment added to the cube of the height divided by twice the chord.



Thus area of segment A D B $= \frac{2}{3} AB \times CD + \frac{CD^3}{2AB}$

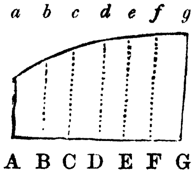
NOTE.—If the segment is greater than a semicircle, find the area of the remaining segment and deduct from the whole circle.

X. *The Ellipse* (see Definition No. 37, page 5).

RULE.—Circumference = mean of transverse and conjugate diameters multiplied by 3·1416.

RULE.—Area = product of transverse diameter multiplied by conjugate diameter multiplied by ·7854.

XI. *The Area of a Segment of a Circle, or any other curvilineal figure by equidistant offsets or ordinates.*



RULE.—If a straight line A G be divided into any number of equal parts, A B, B C, C D, &c., and at the points of division perpendiculars or offsets be erected, A *a*, B *b*, C *c*, &c., to the curve *a b c d e f g*; then to the sum of the first and last offsets, add four times the sum of all the even offsets, and twice the sum of all the odd offsets, not including the first and last; multiply the sum by the common distance of the offsets, and one-third of the product will be the area, recollecting that the second, fourth, &c., are the even offsets, and the third, fifth, &c., are the odd offsets.

If any portion of the figure is not included by an even number of offsets, its area must be found separately and added to the area found by the rule. This rule is known as "Simpson's rule."

XII. *Solid Rectangular Bodies.*

RULE.—Solidity = length, multiplied by breadth, multiplied by height.

XIII. *Cylinders and Prisms.*

RULE.—Solidity = area of base multiplied by perpendicular height.

XIV. *The Prismoid* (see illustration to Definition No. 32).

RULE FOR SOLIDITY.—To the sum of the areas of the two ends add four times the area of a section parallel to

them and equidistant from each; multiply this sum by the perpendicular height and one-sixth of the product will be the volume.

XV. *Sphere.*

RULE.—Solidity = diameter $^3 \times 0{\cdot}5236$

XVI. *Irregular Solids.*

RULE FOR SOLIDITY.—Divide the figure into any even number of parallel and equidistant sections. Find the area of each section. Then add together the first area, twice the sum of all the other odd areas, and four times the sum of all the even areas. Multiply the sum by one-third of the length between each section.

To find the Specific Gravity of a Substance.

Let W = weight of body in air.
Let w = weight of body in water.

RULE.—Specific gravity $= \dfrac{W}{W - w}$.

If the substance be lighter than water sink it by means of a heavier substance and deduct the weight of the heavier substance.

Estimation of Weight from Specific Gravity.

Weight of a cubic foot in lbs. = specific gravity $\times$ 62·425.

CHAPTER III.

GENERAL PRINCIPLES OF SURVEYING.

Surveying is defined in the "Imperial Dictionary" as follows:—"The act of determining the boundaries and area of a portion of the earth's surface by means of measurements taken on the spot; the art of determining the form, area, surface, contour, &c., of any portion of the earth's surface, and delineating the same on a map or plan."

A Plan is defined by the same authority to be "properly the representation of anything drawn; a plane, as a map or chart. . . . . The term plan may be applied to the draught or representation of any projected work on paper, or on a plane surface, as the plan of a town or city."

Operations of the Surveyor.—In any survey, the fundamental principles are the same. There are three distinct operations:—

(1) The taking of the measurements on the ground.
(2) The protracting of such measurements on paper.
(3) The ascertaining, from such protracted measurements, the areas or other specific objects for which such survey was made.

In proportion to the range of the survey these operations vary in magnitude.

In small surveys, simple instruments, simple drawings, and simple calculations only are required; with the increasing scope of the survey, instruments advanced to a higher stage

of accuracy, systems of working more elaborate, and calculations more abstruse, become necessary.

Measurements may be taken either by linear or angular instruments only, or by a combination of both.

Whatever system of measurement is adopted, there is one rule imperative to all. These measurements must be checked and their accuracy proved by means of other measurements, either linear or angular. Only in the case of offsets (explained hereafter) is anything to rest on a single measurement, and if gerat accuracy is required supplementary measurements in this case also must be taken.

The attainment of approximate accuracy in surveying operations is a comparatively simple matter provided a suitable method is employed for the particular job in hand. In many cases this approximate degree of accuracy serves the purpose for which the survey is required, and is such as can only be expected if the surveyor's remuneration depends upon the amount of ground covered. To attain great accuracy the surveyor must exercise great skill, and the expenditure of time is inevitable. Absolute accuracy is unattainable, but, with modern instruments in skilful hands, can be so nearly approached as to serve all practical purposes.

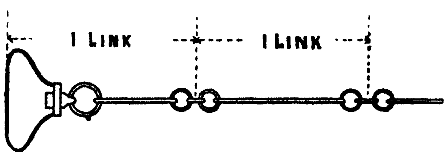
In the last few years great progress has been made in the manufacture and design of levels, theodolites, and other instruments for ordinary surveying purposes. A small modern instrument is at least as accurate as its forerunner of twice the weight and size The permanency of the adjustments, together with ease and convenience of manipulation, have all been studied to such purpose that many of the old patterns which have been practically standardised for many years are rapidly being displaced. In this connection the internally focussing telescope may be mentioned. The power and definition of one of these telescopes by a good maker is altogether so superior to that of the older patterns of approximately twice the size as to be a revelation to the man who has only used the latter.

CHAPTER IV.

CHAIN SURVEYING.

Chain Surveys.—Surveys of the simplest class are known as "chain surveys." This term, although not strictly accurate, inasmuch as the chain is used to supplement the more complex system of measurements, is in general use to denote a survey made without the assistance of angular instruments, but sometimes also with simple instruments of that description—such as the prismatic compass, the box sextant, and the optical square.

Gunter's Chain.—This chain is said to have been invented in 1620 by Edmund Gunter, Professor of Astronomy at Gresham College. The best chain is constructed of thick steel wire, but it can be had in iron wire. It is 66 feet, or 4 poles, in length, and is divided into 100 parts of one link each, consequently equal to $\frac{66}{100}$ths of a foot, or 7·92 inches. These parts consist of a straight length and three oval rings, the measurement of one link being read



from the centre of the middle ring of the one part to the centre of the middle ring of the next part. The handles of the chain are of brass, and are connected by a swivel joint to the wire. The first link is measured from the extremity of the handle to the centre of the middle of the first three rings.

The accompanying sketch will show the method of reading.

At every 10th link from each end of the chain is attached, by a steel ring, a brass counter; that at the 10th link is oval in shape, that at the 20th has two notches, that at the 30th three notches, that at the 40th four notches; and the centre of the chain, or the 50th link, is distinguished by a circular counter. Hence any distance on the chain can be easily ascertained.

The 25th link from each end is constructed differently from the other links, and has a swivel joint. These swivel joints are inserted to prevent the parts "kinking" or becoming twisted, and at the 25th link from each end to facilitate the reading of the chain.

A set of ten steel pins (called arrows), 15 inches long, is supplied with the chain for marking the distances measured. The arrows should be sufficiently strong to enter a macadam road without bending, and it is better to procure what are known as spindle arrows.

The Chain of 100 feet.—This chain is divided into 100 parts of one foot each, and the construction is identical with that of Gunter's chain.

Comparative Advantages of Gunter's Chain and the Chain of 100 Feet.—For measuring distances on estate surveys, where the computation of acreage is one of the main objects, the Gunter's chain is more convenient, as an acre contains 10 square chains, or 100,000 links, and the content, as ascertained in square chains and links, can be readily reduced to acres and decimals of an acre merely by dividing by 10, thus avoiding the more laborious calculations necessary for reducing superficial feet.

In plans required by Parliament for statutory undertakings, when the distances are to be stated in miles and furlongs, Gunter's chain is also the more convenient, from its being $\frac{1}{80}$ of a mile and $\frac{1}{10}$ of a furlong.

The chain of 100 feet is better adapted for surveying in towns, where widths of streets are required to be stated in feet and inches, and also when distances have to be taken in connection with levelling operations, as the measurements in this case, both vertical and horizontal, are set down in feet and fractions of a foot.

In a given distance, also, the chain of 100 feet requires to be stretched less frequently than the Gunter's chain, and

as there is in chain surveying, however carefully done, a certain inaccuracy in the measurement each time a chain length is measured, the inaccuracy in the total measurement will be the greater when Gunter's chain is used.

The Tape.—This is made of steel, or of linen painted and varnished. It can be had in various lengths, from 33 feet to 100 feet, but the tape of 66 feet in length is more commonly in use. On one side the tape is divided into feet and inches, and on the other into poles and links. The steel tapes are of a standard measurement, at a temperature of 62° Fah., and are more reliable than the linen tapes, which stretch with use in dry weather and shrink after getting wet. A linen tape should be used with great care, and only in fine weather. The steel tape is very liable to twist during use, and consequently is very easily broken, and to be properly repaired has to be sent to the maker, but with careful use it will last a long time.

Poles are used to mark the commencement, intermediate points in, and end of the survey lines, and are made of yellow pine, shod at one end with wrought iron or steel, brought to a point. They are circular in section, and $1\frac{1}{8}$ inch in diameter, tapering at the top to 1 inch. The most convenient size is 8 feet in length, including the shoe. They are painted in alternate colours, black and white, or red and white, and preferably in lengths of 2 feet.

Flags.—When the survey is large and the extremities of the lines at a considerable distance from each other, it is often extremely difficult to distinguish the outline of the poles. Small red and white flags are used for tieing to the poles in such a case, and, moving in the wind, render the desired point of view more easily discernible.

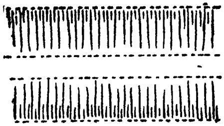
The Offset Staff, used for the purpose of measuring offsets (see p. 37), is similar to a ranging pole, and usually 10 links in length. Each link is painted white, with a black ring at the extremity of each link. Instead of the pointed shoe, the offset staff is sometimes supplied with a hook at the end. This, it is said, is for convenience in dragging the chain through a fence, though a result with more precision can be obtained by placing the ring next the handle of the

chain on the point of an ordinary ranging pole, and pushing it through the fence. In the author's opinion an offset staff specially constructed is not required, as in most cases a ranging rod, 8 feet long, painted as described in lengths of 2 feet (or 3 links nearly) is sufficient for taking offsets as accurately as desired.

Comparison of Chains and Tapes with Standard Measurement.—Before commencing a survey the chains and tapes to be used should be compared with their standard measurement, and, if the survey lasts more than one day, then this comparison should be made daily. This standard measurement can be obtained from a standard chain kept solely for this purpose; and it is also conveniently obtained by a surveyor setting out on the curb-stone of the footpath in front of his office the standard lengths of one chain of 66 feet and of a chain of 100 feet, placing notches in the stone at the extremities of such distances. In case the survey is at some considerable distance from the office, the surveyor, having ascertained that his chain is of standard measurement, should set out this length in the district where the survey is being made, similarly to that in front of his office.

In the event of producing measurements in a court of law this proceeding is imperative (see the Reports of the Commissioners for the Restoration of Weights and Measures), as the chain is composed of parts not welded together, which sometimes open out after use, and sometimes become bent, as in passing through fences. In wet weather also the chain gets shorter by dirt accumulating between the rings. Should the chain be too short as compared with the standard measurement, it will be found to arise from bent links, which should be hammered out straight; and if it be found to be too long, one or two rings must be removed from the chain. Any such removal of rings must be effected equally on each side of the centre of the chain, so that the fiftieth link may occupy its true position.

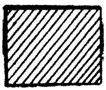
Descriptive Symbols used in Surveying.—The following are symbols commonly in use :—



Cutting.

Highway.

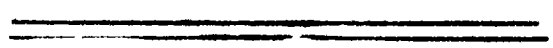
Fence (ditch shown by mark T).



Building.

Plantation.

Private Road (unfenced).

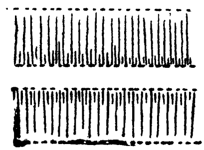


Railway.

Fence.

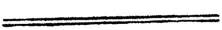
Line of Curb Stones.

Gate.



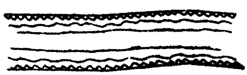
Embankment.

Rail Fence.



Wall.

Footpath.



River or Watercourse.

The Field Book.—The sketches and measurements taken on the ground, when surveying, are entered at the time in a "field book," which should be paged for future reference. The page is usually $7\frac{1}{2}$ inches long and 4 inches wide. Two kinds of field books are in use among surveyors—one having one line down the centre of each page lengthwise, and the other having two lines distant apart about a quarter of an inch.

The line (or, in the second case, lines) represents the chain-line as set out on the ground, and is reserved for distances measured on it, at which observations are made, such as points at which hedges are crossed, or offsets taken, stations noted, &c. On the spaces in the field book, right and left of this line, those offsets and any other necessary observations must be entered, according as they are situated on the right or the left of the chain-line. The first measurements are entered at the foot of the page, and the others follow in succession towards the top.

The field book with one line for the chain-line is the more convenient, and is now in more general use. It gives more space for the sketching, and the marking of fences crossing the chain-line and the intersection of survey lines are less confusing and more easily drawn.

In the field book with the double column, any fence meeting the chain-line at an angle must not be drawn across the column in the field book, but be drawn on one side of the column and start from it on the other side at a point precisely opposite, as if the chain-line were of the thickness of the column.

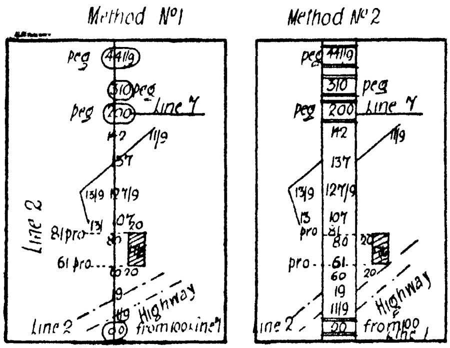
The diagrams here given (on opposite page) represent the two methods of entering identical measurements in the field book.

All sketching should be neatly done and be accurate and well proportioned, the lines being firmly drawn.

Each survey line should be begun on a fresh page. In case it should be necessary to add or subtract any figures for the purpose of the survey—such as often occurs in ascertaining the intersection of survey lines—the figures should be entered at the side of the page, and the work shown in full in the field book.

Buildings are usually surveyed by taking offsets (page 37) to the corners opposite the chain, and noting the points on the

chain at which the sides of the buildings, bounded by these points, would meet, if produced. In the figure an offset of 20 feet is taken from 60, and the side of the building produced would meet it in 61. In town surveying, however, this method is not sufficiently accurate, and measure-



ments must be taken from two convenient points on the chain-line to the corner of the building, thus forming a small triangle. The production of the building on to the chain-line should be noted, as in the former case.

Method of Chaining. — Two assistants, termed the "leader" and the "follower" respectively, are required by the surveyor to perform the measurement. The leader takes the handle of the chain in his left hand, and the chain itself in his right, and throws it from him in the direction in which the measurement is to be taken. He then goes to the centre of the chain and sets straight any twisted links, and starts with the ten arrows in his left hand and one end of the chain in his right, along the required line, while the follower remains at the starting point, and looking at the poles in front of him that mark the line to be measured, directs the leader into line by signs with his left hand. The leader takes an arrow in his right hand, holding it perfectly vertical and low down, his hand being practically on the surface of the ground, to insure steadiness, places the arrow against the end of the chain, and, when in perfect alignment, is told by the follower to

"mark," and fixes it firmly in the ground. The chain must now lie so that offsets and other measurements can be taken from it. It should be examined as it lies, to see that no links are bent or twisted; great care should be taken that the chain is perfectly straight and tightly stretched, that the arrow is coincident with the end of the chain, and is marked steadily. Inexperienced chainmen, when receiving the order to mark, often do so with a jerk, setting down the arrow a distance of one or two inches from its true position. The strictest accuracy is required when chaining.

When all measurements on the length of the first chain are taken, the leader proceeds a second chain's length in the same direction, while the follower comes up to the arrow first put down. A second arrow being now put down by the leader, the first is taken up by the follower; and the same operation is repeated till the leader has expended all his arrows. Ten chains, or 1,000 links having now been measured and noted in the field book, the follower returns the ten arrows to the leader, and the same operation is repeated as often as necessary.

The arrows on being exchanged must always be counted, both by the leader and the follower, and this observation applies also to the commencement of the measurement. When the leader arrives at the end of the line, the number of arrows in the follower's hand show the number of chains measured since the last exchange of arrows noted in the field book; and the number of links extending from the last arrow to the pole at the extremity of the line, being also added, give the entire measurement of the line. Thus, if the arrows have been exchanged seven times, and if the follower have six arrows, and from the arrow last put down to the end of the line be 83 links, the whole measurement will be 7,683 links, or 76 chains 83 links.

$$\begin{array}{r} 7000 \\ 600 \\ 83 \\ \hline 7683 \end{array}$$

In using the chain, care must be taken to compare it with the standard measurement, as previously pointed out, as it will extend by use, and will therefore require to be shortened, but a good chain may be used for several days without any material extension.

The chain is folded in four links at a time, viz., two on each side of the centre, at which point the folding is commenced. Mistakes occasionally arise in the reading of the

chain, owing to the brass counters being identical on each side of the centre, at their respective distances from it—the counter 40 being read for 60, or *vice versâ;* the counter 70 for 30, and so on. It is always well, therefore, in reading a station to read the chain twice, identifying at the same time the position of the centre of the chain.

Measuring Sloping Ground.—The term "plan" has already been defined (p. 21). In other words, a plan is a horizontal representation of the features of the ground. All measurements must therefore be reduced to a horizontal plane. Any undulations of the ground can be shown on what is known as a "section," which will be explained in the chapter on Levelling. A plan is, therefore, not the actual surface of the ground, but the diminished quantity that would result were the whole projected on a horizontal plane. In rising or falling ground the slope represents the hypotenuse of a right-angled triangle, and is consequently of greater length than another side of the triangle, viz., the horizontal distance.

The sloping measurements can be reduced to horizontal by calculation, based on the angle of the slope with the horizon, which, in such a case, would be observed with a clinometer, or other angular instrument. These calculations are laborious, and as it seldom happens that the ground slopes in one angle with the horizon, each different angle of slope would have to be observed, and a separate calculation made for it. If the slope be not very steep, the horizontal measurement can be obtained, by holding up, horizontally, as nearly as can be judged by the eye, a part of the chain, say 25 links at a time, and allowing a pointed plummet to mark the measurement on the surface, this operation being continued until the undulating ground is measured. This method is known as "stepping," and is recommended as being generally sufficiently accurate for small surveys. The steeper the slope, the less the length that can be measured at each holding up of the chain.

Another method is to make an allowance, at the time of measuring, for the slope according to the following table, the angle of slope being judged by the surveyor, and the arrow set back the number of links in the table corresponding to the particular angle of slope.

Table

showing the reduction in links and decimals of a link upon 100 *links for every half-degree of inclination from* 5° *to* 20° 30′.

Angle.	Reduction.	Angle.	Reduction.	Angle.	Reduction.
5° 0′	0·38	11° 0′	1·84	16° 0′	3·87
30	0·46	30	2·01	30	4·12
6 0	0·55	12 0	2·19	17 0	4·37
30	0·64	30	2·37	30	4·63
7 0	0·75	13 0	2·56	18 0	4·89
30	0·86	30	2·76	30	5·17
8 0	0·97	14 0	2·97	19 0	5·45
30	1·10	30	3·19	30	5·74
9 0	1·23	15 0	3·41	20 0	6·03
30	1·37	30	3·64	30	6·33
10 0	1·52				
30	1·67				

By this Table the trouble of computation is avoided, only the distance measured on each rise or fall requiring to be multiplied by the reduction in chains corresponding to the angle of each rise or fall; and the product, subtracted from that distance, will give the correct distance.

Example. — A line was measured 12·43 chains, on ground having a rise of 8½ degrees; required the horizontal length of the line.

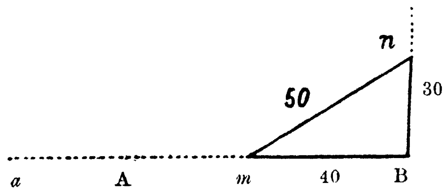
Here to 8½°, or 8° 30′, corresponds the reduction 1·10 links, whence

12·43	Whence 12·43
1·10	13·673
13·6730 links	12·29·327 horizontal distance,

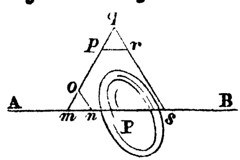
in which the decimal, being less than half a link, is rejected; thus making the correct horizontal distance 12·29 chains, or 1,229 links.

To Erect a Perpendicular with the Chain.—Let *a* B be a chain-line, and A B the extended chain. It is required to erect a perpendicular to *a* B at B. Fix the end of the chain to the ground with an arrow at B; fix also

the 80th link of the chain, reckoning from B, at *m*, 40 links from B; 80 links of the chain now lying slack between B and *m*. Take hold of the 30th link of the chain from B, and extend it till it takes the position B *n m*, the portions B *n*, *m n* of the chain being pulled tight; then shall B *n* be perpendicular * to the chain-line *a* B, and may be extended to any length required. This method is easily remembered from the fact of the sides of the triangle being multiples of the figures 3, 4, and 5.



To Measure a Line impeded by an Object not obstructing the Sight.—Let A B be a chain-line, the direct measurement of which is prevented by the unforeseen obstruction of a pond, P.



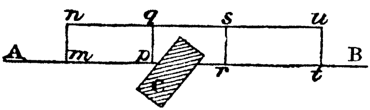
Measure A *n* till it reach to, or near to the edge of the pond, and fasten the ends of the chain to the ground with arrows at *m* and *n*, the distance *m n* being made half a chain or 50 links. Take hold of the middle of the chain, and extend it firmly, till its two halves rest in the positions *m o*, *o n*; thus making an equilateral triangle *m n o*, each side of which is 50 links. In the direction *m o*, measure to nearly opposite the middle of the pond, as to *q*. Again, make *p q* equal 50 links, fasten the ends of the chain at *p* and *q*, and extend its middle point to *r*, as before. In the direction *q r*, measure to *s*, till *q s* be equal to *m q*. Then *s* will be in the line A B, and *m s* † will be equal to *m q* or

* Since the parts B *n*, *n m* of the chain are together 80 links, of which B *n* is 30, the remainder *n m* is therefore 50; also B *m* was made 40; whence $40^2 + 30^2 = 50^2$, that is $m\,B^2 + B\,n^2 = m\,n^2$, therefore by Euc. I. 47, B *n* is perpendicular to B *m*, or *m* B *n* is a right angle.

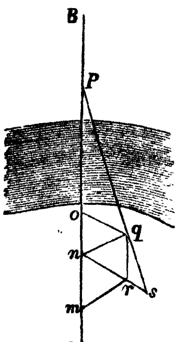
† Because the triangles *m n o*, *p q r*, are both equilateral, the angles at *m* and *q* are each 60° or one-third of two right angles; whence by Theorem IV. the angle at *s* is also 60°; therefore all angles of the triangle, and consequently its sides, are equal, that is, $m\,q = q\,s = m\,s$.

$q\,s$, which being added to A m will give the distance A s. Offsets being taken to the margin of the pond, during the measurement of the lines $m\,q$, $q\,s$, the measurement from s to B may be continued.

To Measure a Line impeded by an Object obstructing the Sight, as a Building.—Let A B be a chain-line, the measurement of which is prevented by the building C, but on which points can be set out in line with B as far as the side of the building nearest A.



At m, 4 or 5 chains from the building, take a perpendicular $m\,n$, of such a length that the line $n\,s$ may clear the building B. At or near the building take another perpendicular $p\,q$, exactly equal to $m\,n$ (these perpendiculars ought to be measured with the chain or tape), and poles being put up, truly vertical at n and q, measure $q\,s\,u$ in the direction $n\,q$ of the poles, taking offsets to the building till it be cleared at s. Now on the line $q\,u$, at the distance $s\,u$, at or about equal to $m\,p$, erect the perpendiculars $s\,r$, $u\,t$, each exactly equal to $m\,n$ or $p\,q$, fixing poles, truly vertical, at r and t. These poles are in the true direction A B, and the measurement of the line may now be continued from r to B, after adding the distance $q\,s$ (which is equal to $p\,r$) to A p.



If the building or other object only protrude a few links over the line, the perpendiculars $m\,n$, $p\,q$, $s\,r$, &c., may be erected by the offset-staff, as nearly correct as can be judged by the eye, and the results should be sufficiently accurate. *

To find the Width of a River which is too wide to be measured across with the Chain.—Let A B be the chain line crossing a river, situated between o and p, two points on the chain line and near the river on opposite sides. A pole being fixed at p, on A B lay off $o\,n$, $n\,m$, each equal 50 links and with the ends of the chain suc-

* See note at end of Chapter.

cessively fixed at o, n, and at n, m, lay down the equilateral triangles $o\,q\,n$, $n\,r\,m$, as previously shown, poles being fixed at n, q, and r. In the two directions $p\,q$, $n\,r$, fix a pole at s, and measure the distance $r\,s$ accurately with the tape to one-eighth of a link. Then by the similar triangles $s\,r\,q$, $q\,o\,p$, we shall have $r\,s : q\,r :: o\,q : o\,p$.

But $q\,r = o\,q = o\,n =$ 50 links, therefore, $r\,s : o\,n :: o\,n : o\,p$; therefore

$$o\,p = \frac{o\,n^2}{r\,s} = \frac{50^2}{r\,s} = \frac{2500}{r\,s}.$$

And $r\,s$ having been carefully measured the distance $o\,p$ becomes known.

Rule.—Divide 2500 by the distance $r\,s$, and the quotient will be the breadth of the river, or the distance $o\,p$, which must be added to A o to give the distance A p.

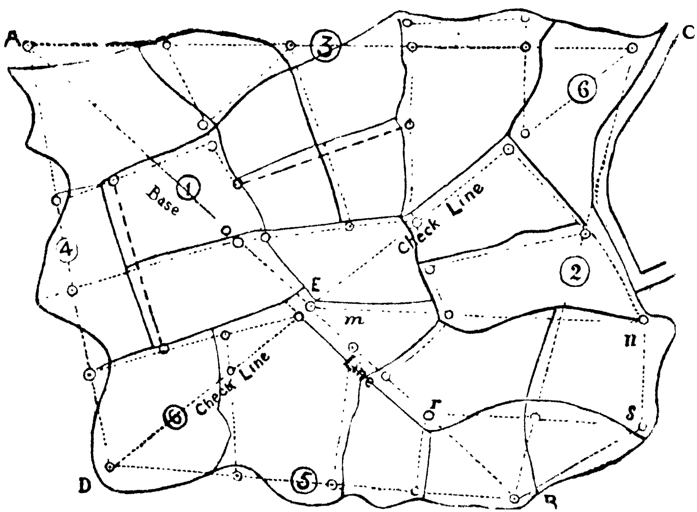
Example.—Required the breadth of a river by this method, when $r\,s$ measures 15 links.

$$\text{Answer} = \frac{2500}{15} = 166\tfrac{2}{3} \text{ links.}$$

Theory of Triangulation.—The definition of "triangulation," according to the "Imperial Dictionary," is "the act of triangulating, the reduction of the surface of an area to triangles, for purposes of a trigonometrical survey." Triangulation is also the primary principle of chain surveying.

In a trigonometrical survey, one side of a triangle is measured, and the angles included between this side and the other sides of the triangle observed. The length of these sides can then be computed by trigonometry. In a chain survey the three sides are measured. In both cases, from these data the triangle can be laid down on paper. The principle is enunciated in Euclid, I. 7 (see Theorem I., page 5), which sets forth that "upon the same base and on the same side of it, there cannot be two triangles that have their sides which are terminated at one extremity of the base equal to one another, and likewise those which are terminated at the other extremity equal to one another." From this it will be seen that, the sides being constant, the form and position of the triangle are fixed and unalterable.

Practical Application of Chain Triangles. Survey Lines.—The area to be measured is included within an imaginary triangle, or a series of imaginary triangles. The lines forming the sides of the triangle or triangles are measured with the chain, and, as well as others so measured, are known as "survey lines." All survey lines should pass as near the objects or boundaries of the area to be surveyed as practicable, so as to avoid the use of long offsets, and so as to include the whole, or as much as possible, within one large triangle or a system of large



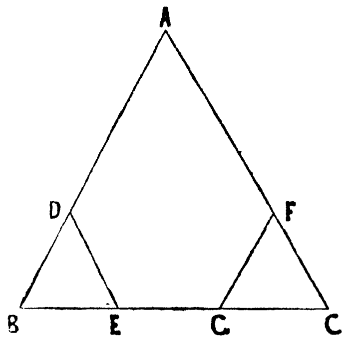
triangles. The base of the triangle, or of the main system of triangles, should be arranged to extend throughout the whole length of the survey, wherever circumstances will allow it. This line (or base) is called the "base line."

Care should be taken in setting out the triangles to form, so far as the nature of the ground will admit, well-conditioned triangles—that is, not having very acute or obtuse angles, and the sides consequently not very unequal. The nearer to the equilateral form the better the triangle, and, in the case of an error of measurement in one of the sides,

the less the alteration of the figure of the triangle from the true form.

Check Lines.—The skeleton lines of the survey having been measured, it is necessary to prove their accuracy by means of further measured lines, termed "check" or "tie" lines, and this is best attained by lines measured from the apex to the base of each triangle. These lines should, if possible, form an angle with the base of from 60° to 90°.

Where lines from the apex are not available, owing to the nature of the ground, the triangle may be checked by lines measured across the adjacent sides of any two angles as the lines D E and F G in the triangle A B C.



Filling in. Offsets.—The fences, surface details, and features of the ground included within the skeleton of the survey, are measured by "offsets" from lines run within the interior of the triangles known as filling-in lines. An offset is the distance measured at right angles from the chain to the point which it is desired to reproduce.

As stated above, the survey lines should be so arranged that the offsets may be as short as possible. Generally speaking, a limit of 20 to 30 links should not be exceeded, except for plans to be drawn to a small scale, such as six chains to the inch. On the Ordnance Survey the offsets were limited to 20 links for the $\frac{1}{2500}$ map, or map of 25·344 inches to the statute mile, and to 80 links for the $\frac{1}{10560}$ map, or map of six inches to the mile.

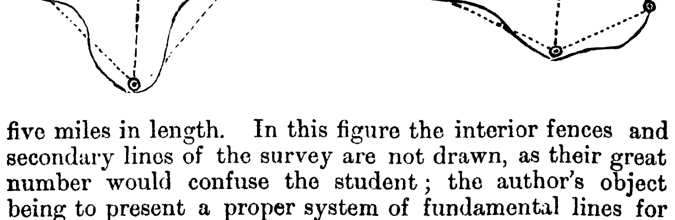
The main filling-in lines should be run between fixed points on two of the skeleton lines of the survey, and all filling-in lines must be self-checking when laid down on paper. The check lines can often conveniently be set out as filling-in lines.

Diagrams to Illustrate Chain Surveys.—In the

diagram on page 36, the survey lines are shown by dotted lines; and to make the matter more easily intelligible, the base line, the sides of the main triangles, and check lines only, are numbered, although in practice the filling-in lines would also be numbered. It will be noticed that lines 2, 3, 4, and 5 pass for some portion of their length into lands adjoining that which it is required to survey.

The base line is measured from A to B, line 2 from B to C, and line 3 from C to A. This completes the first triangle, which, however, is not yet checked. Line 4 is measured from A to D, and line 5 from D to B. This completes the second main triangle. A check line is now measured from D to C, crossing the base line in E, verifying the two triangles. It will be seen that the lines *mn*, *rs* are prolonged beyond the system of main lines to survey the part on the right of line 2. It happens on occasions that the shape of the ground is such as to prohibit, without excessive labour, the whole area being included within one or more triangles. In such cases the lines along the boundaries must be run between points determined by chain triangles, and the framework will then probably consist of a combination of triangles and trapeziums.

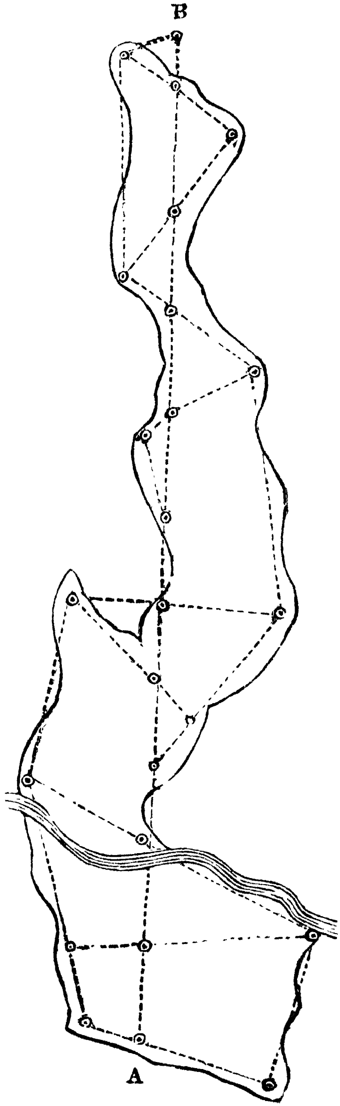
The system of main lines adopted by the author in the survey of the parish of Woolbeding, in the county of Sussex, was the following, the base line A B being about



five miles in length. In this figure the interior fences and secondary lines of the survey are not drawn, as their great number would confuse the student; the author's object being to present a proper system of fundamental lines for the survey in question.

The parish of Lodsworth, in the same county, presents

another variety of shape. The base line A B (see figure below) was nearly seven miles in length.



Inspection of Ground. Setting Out.—In order to ascertain the best position and obtain a clear course, as free from obstacles as possible for the survey lines, it is first necessary to walk over and examine the ground. If the survey is large, a tracing from the 6″ or 25″ Ordnance Map will be useful, on which to mark the intended survey lines. In the event of no tracing from an existing map being available, the surveyor should make a rough sketch of the area, and mark his lines on it. Conspicuous objects, as, for instance, the corner of a church tower, on which lines can be directed if required, should be noted, and all lines should be ranged out perfectly straight, the poles being set up exactly perpendicular. Should the lines be of considerable length, they may be set out by means of a field-glass, care being taken that the axis of the instrument is in true alignment.

Stations. Pegs.—The commencement of a survey line, its junction with

another line, and any other points the surveyor may desire to fix on the chain line on the ground, are known as "stations," and are conventionally shown in the field book by an oval sign surrounding the measurement, thus (2225). A peg should be placed at each station, and in case a pole is required at the station, it must be set up behind the peg. The peg should on no account be removed. Pegs may conveniently be 15″ long, 1¼″ square, and sharpened at one end. They should be driven into the ground so that the top of the peg may be about 1¼ inches above it.

General Observations.—When a survey is made for a finished plan, all buildings, roads, rivers, ponds, footpaths, gates, fences, and the like, should be noted down. All survey lines should be numbered consecutively, the base line being line 1 where possible, and the direction of the north with reference to it should be taken with a pocket compass, and noted in the field book. When all the lines forming the skeleton of the survey, and the tie lines, have been measured, they should be plotted on paper, in order that their accuracy may be checked, before the filling-in lines are proceeded with.

On an extensive survey, more than one surveyor with his assistants can with advantage be employed, each surveyor taking a separate portion of the triangulation, and being furnished by the chief surveyor with a copy of his field book relating to the main lines on which the particular portion abuts.

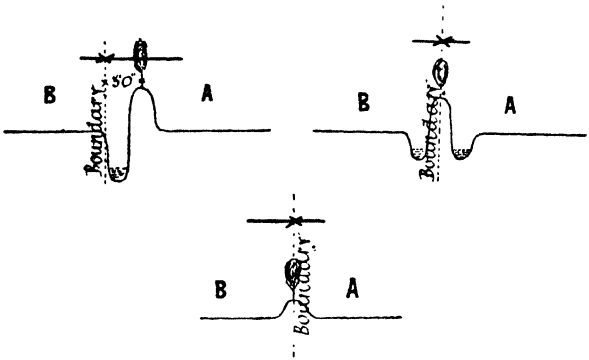
Boundaries.—The boundary of the estate measured ought to be carefully observed, and enquiry made with respect to it, as custom varies in different parts of the country.

All measurements should be taken to the centre of the roots or "quicks" of the fence, and an allowance made for the ditch when estimating the area. In some parts of the country an allowance is made of 4½ links (or 3 feet), and in others of 6 links.

The following illustrations show the cases of boundaries commonly met with.

In the first illustration A's boundary is the distance allowed by custom, 3 feet in this particular part of the country (Lancashire), from the centre of the quicks to

include the ditch, the soil from the ditch being excavated to form a "cop" or bank on which to plant the thorns. In



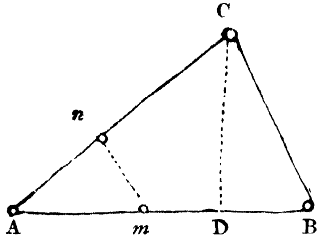
the other illustrations the boundary is the centre of the quicks.

On an estate plan the fences belonging to the estate are usually marked thus ——T——, the mark T indicating the position of the ditch.

Simple Surveys.—In the examples given below, a field book with two columns for the chain-line is used, as examination questions are often stated in that way. In ordinary practice the field book with one line is more useful, as explained on page 28. For the same reason the long offsets appearing in some of the examples are inserted, although in practice, as already stated, they should not exceed 30 links, except for plans drawn to a small scale.

I. *Triangular fields.*—When a triangular field is to be surveyed, set up poles at each corner, and measure lines near each side, taking the offsets and leaving stations in at least two of the lines. Enter their distances in the field book, and measure the distance between the two stations for a proof-line. Where no obstacle to prevent direct measurement intervenes, one only may be left in one of the lines, which may be connected with its opposite angle for a proof-line, as the line C D.

Example.—Required the plan of a field from the following dimensions :—



	to ⊙ *n*	
Proof	384	line
From	⊙ *m*	
	to ⊙ A	
	1244	
⊙ *n*	700	
	L. ⊙ C	
	to ⊙ C	
	852	
	L. ⊙ B	
	to ⊙ B	
	1338	
	1000	
⊙ *m*	600	
From	⊙ A	range E

When the triangle A B C is constructed, the proof-line *m n* will be found to measure 384 links, if there has been no error in the work: but if when measured from the plan it does not exactly, or within the limit of 1 link, agree with that measured in the field, some error has been made, and the work must be repeated.

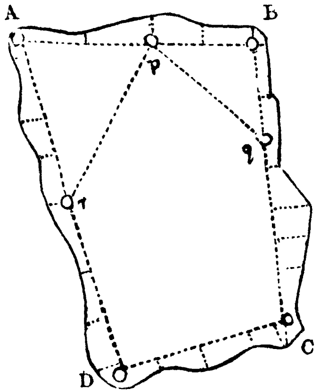
II. *Four-sided fields.*—When a field has four sides, measure lines near them, taking the offsets; also measure both the diagonals, one of which will serve as a base in plotting the work, and the other for a proof-line; or the proof-line may be measured in any other direction that may be most convenient.

Sometimes the measurement of both the diagonals is prevented by obstructions. In such cases it will be sufficient to measure tie-lines across two of the angles of the trapezium, at the distance of from two to five chains from each angle, according to the size of the field. These tie-lines, with their distances from the angles on the main lines, will be found sufficient for planning the lines and proving them in small fields. But when the main lines that include the chief part of the ground to be measured are of considerable length, as from 30 to 40 chains, it will be necessary to take the tie-lines at least 10 chains from the angles, across which they are measured; for a small error, in laying down the

plan with short tie-lines, will cause the main lines to deviate considerably from their true position when prolonged.

However, it sometimes happens that long tie-lines cannot be obtained, in consequence of obstructions. In such cases the tie-line must be carefully measured to even one-fourth of a link; the distance of each tie-line from its angle and the tie-line itself must then be all multiplied by four, thus throwing fractions out of the question, and with the three lines, thus increased, the triangle determining the position of an angle of the trapezium may be accurately constructed.

The proof-line and its distances from its angle must be similarly treated, that the accuracy of the work may be fully established.



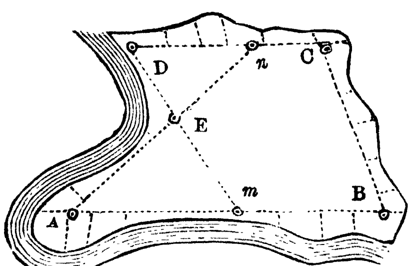
Example 1.—In the annexed figure the lines A B, B C, C D, D A are measured, stations being left at p, q and r, and their respective distances on the lines noted in the field book, thus furnishing the following method of laying down the plan.

On A B, as a base, take A p = given distance, and with the distances A r, p r, and centres A and p describe arcs cutting in r; then prolong A r, and lay off thereon the given length A D. In the same manner construct the triangle p B q, and make B C = its given length. Lastly, join D C, which must be of the length shown in the field book, otherwise there has been some mistake, either in the measurement or in laying it down. Should this be the case, the whole of the work, firstly on the plan, and secondly in the field, must be gone over again till the error be discovered.

Example 2.—Required the plan of a straight-sided field from the following dimensions:—

	to ⊙ A	
	952	
To ⊙ n 481 Proof-line .	452	⊙ m
	L. ⊙ D	
	to ⊙ D	
	1236	
To ⊙ B 833 Proof-line .	400	⊙ r
	L. ⊙ C	
	to ⊙ C	
	886	
	L. ⊙ B	
	to ⊙ B	
	1446	
⊙ n	500	
From	⊙ A	go North.

III. *Fields having more than four sides.*—Various methods will suggest themselves to the surveyor for taking lines to lay down a field that requires more than four main lines to take its boundary. A few methods only of surveying fields of this kind will, therefore, be presented; although their variety of shape is such that no general rule can be given for laying out, on the ground, lines that will give an accurate plan.



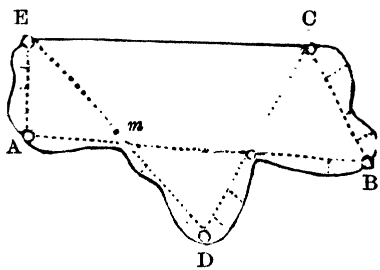
Example 1.—Here a field of five sides is surveyed by the same number of lines, viz., A B, B C, C D, D m, and A n, the last two intersecting in E. These lines evidently constitute a decisive proof among themselves, and all of them are available in taking the boundary.

In surveying this field (poles being fixed at A B C D

and E), commence close to the river's edge, in the line A B prolonged backwards, enter the offsets and the station A in the field book. On arriving at ⊙ *m*, in the direction E D, enter its distance, and so on to ⊙ B, measuring the line to the fence; from B proceed to C, in like manner, measuring beyond the station to the fence. The place of the ⊙ *n* is to be noted, on arriving in the direction E A, while measuring C D. D *m* is next measured, the place of the ⊙ E being noted. Lastly, measure A *n*, entering the place of the ⊙ E a second time, all the offsets being supposed to be taken during the operation.

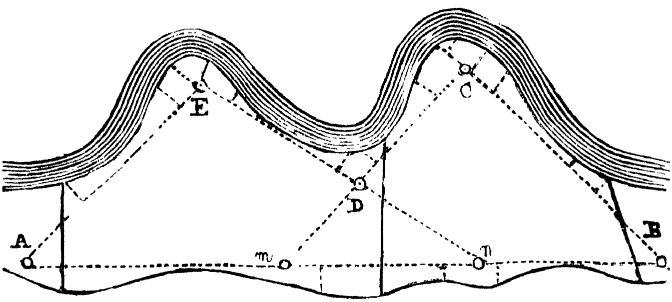
Construction of the plan.—Select the distances A *m*, A E and E *m* from the field book, and with them construct the triangle A *m* E, prolong the sides to their entire lengths, up to the boundaries, and fix the places of the stations B *n* and D. Now, if the measured length of D *n* just fits between D and *n*, the work is right with respect to the triangles A E *m*, E D *n*. Lastly, prolong D*n* to the ⊙ C, and if the distance from thence to the ⊙ B be the same as shown by the field book, the whole of the work is right. But if the distance D *n* does not agree, the work must be examined from the beginning; if only the distance B C fails, then only that distance and the portions *m* B, C *n* need be examined.

Example 2.—In the annexed figure, A *m* D *n* B C E is a seven-sided field (or, rather, is resolvable into a seven-sided field for surveying); one of the sides C E is straight, and the fence A *m* D *n* B is too much bent to be taken by one line crossing and recrossing it with offsets taken to the right and left. The lines here required are only six, viz., E C, A B, B C, C D, D E, and E A, which may be measured consecutively. The stations *m* and *n* in A B, being carefully noted in the field book, give at once the means of laying down the plan, and proving its accuracy. To plot the work, lay down the line A B: the triangle *m n* D should be then



laid down, and its sides D *m*, D *n*, prolonged to the stations C and E, from whence the lines C B, E A, must respectively reach the points B and C, and the line C E equal the measured distance to confirm the accuracy of the work. This done, the offsets on the several lines may be laid off, through which the fences are to be drawn.

Example 3.—This figure comprises two fields by the side of a river; each field, for the purposes of surveying, may be considered a five-sided field. The five lines A B, B C, C *m*, *n* E, and E A, are found amply sufficient to accomplish



an accurate survey, in consequence of the three fences terminating at the river being straight; their positions are determined by the intersections of the surveying lines, the middle fence by three intersections, and the two end fences by two intersections and one offset each, thus proving the accuracy of their positions. The two main triangles of the survey, viz., A *n* E, *m* B C, mutually prove one another by the intersection at the station D.

In laying down the figure, the largest triangles ought to be laid down first, and the accuracy of the plan will be shown by the agreement of the ⊙ D with the proper intersection of the lines C *m*, E *n*.

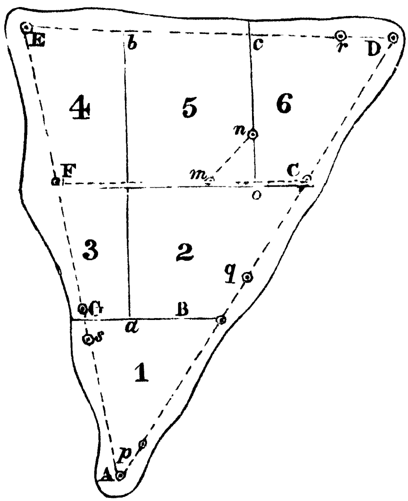
Example 4.—Required the plans of two straight-fenced fields, each having five sides, from the following field notes:

   Return	To $\odot$ A 2083 1000 to $\odot$ D	 diag.
	to $\odot$ E 840 R. $\odot$ C	 diag.
From	to $\odot$ C 1537 1000 R. $\odot$ A	to $\odot$ *m* 519 diag.
	to $\odot$ A 1170 R $\odot$ E	
	to $\odot$ E 1035 R. $\odot$ D	
	to $\odot$ D 730 L. $\odot$ C	
	to $\odot$ C 779 R. $\odot$ B	
    Begin	to $\odot$ B 2107 2000 1200 at $\odot$ A	 $\odot$ *m*. go W

	To $\odot$ B 755 R. $\odot$ E	 diag.
  From $\odot$ *m*	to $\odot$ E 619 200 R. $\odot$ C	 diag. to $\odot$ D 142
	to $\odot$ C 599 R. $\odot$ A	 diag.
	to $\odot$ A 196 R. $\odot$ E	
	to $\odot$ E 346 R. $\odot$ D	
	to $\odot$ D 309 R. $\odot$ C	
	to $\odot$ C 267 R. $\odot$ B	
  Begin	to $\odot$ B 667 at $\odot$ A	 range W.

To Survey a Small Estate, divided internally by Straight Fences.—A small estate, of a form nearly triangular, is divided into six fields by four straight fences, as shown on the annexed plan.

The survey is commenced at A, by measuring the line A D, taking the offsets, and leaving stations at the crossing of these fences at B and C. The line D E is now measured, also crossing two of the straight fences. From the ⊙ E the line E A is measured, leaving stations at the hedge-crossings F and G: thus completing the triangle A D E. Next, the



line B G is measured, close to the straight fence B G, and crossing the straight fence *a b*.

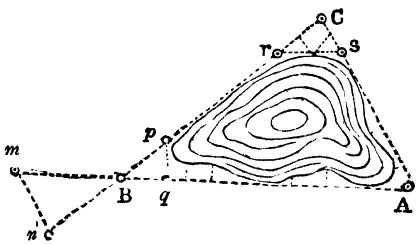
Lastly, the line F C is measured, again crossing the straight fences *a b* and *c o*: a station being left at *m*, about a chain's distance from *o*, and one chain measured close by the fence *o c* to *n*, and from thence to ⊙ *m*: thus completing the survey. The position of the straight fence *a b* is proved to be correct by the crossing of three of the lines of the survey, viz., B G, F C, and D E; the crossings made by these three lines on *a b*, must be in a straight line on the plan, otherwise there has been an error in the field

notes. The straight fence o c has only two crossings by the survey lines F C, D E, its position is, therefore, not duly proved to be correct without the tie-line m n, measured in the manner already stated.

Thus the survey of six fields may be made, and its accuracy proved, by five lines with two short tie lines m n and n o, which may be regarded as mere offsets.

Woods, Lakes, and Swamps.—When woods, lakes, and swamps are required to be surveyed a system of lines must be adopted for each case, so combined by triangulation as to prove their accuracy when laid down on paper. If the wood or other inaccessible space (as far as measuring is concerned) be either triangular or very nearly so in shape, the three sides of a triangle will compass it, which may be proved by tie-lines at its corners, if the wood or lake will admit them of sufficient length; but if not, any two of the sides of the triangle may be prolonged for this purpose, offsets being taken to the boundary of the wood or lake in the usual way.

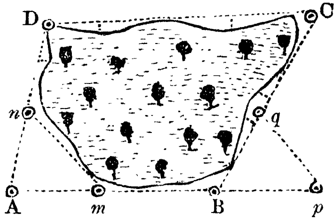
Example 1.—Here the three sides of the triangle A B C enclose a lake or large pond, offsets being taken therefrom to the margin of the water. The accuracy of the work is proved by the tie-lines p q and r s, or, if these lines be too



short, the sides A B, C B may be prolonged to m and n, till B m, B n, each equal about one-third of A B, and the tie-line, m n, being measured, as well as the line r s, will prove the accuracy of the work.

Example 2.—The annexed figure shows the survey of a wood, which is effected by the four main lines A B, B C,

C D, D A, the first line being prolonged to p, and stations being left at m, n, and q for the tie-lines $m\ n$, $p\ q$.



It scarcely need be added that when the lines A B, A D, B C, are laid down by the help of the two tie-lines, the line C D will exactly fit in, if the work has been accurately done.

Example 3.—Required the plan of a wood from the following field notes, two of the fences of the wood being straight.

	To ⊙ p	
	1175	
⊙ C 0	825	
28	600	
0	500	
	400	⊙ n 491 to ⊙ m
0	275	
20	150	
52	25	
0	000	
Return	to ⊙ B	
	to ⊙ m	
	900	
⊙ B 0	600	
25	500	
0	425	
0	300	
50	200	
40	125	
0	000	
From	⊙ A	**go E.**

	To ⊙ A	
0	812	
	⊙ D	
	to ⊙ D	
	575	
	400	⊙ q 550 to ⊙ p
Return	to ⊙ C	

As a general rule for surveying woods and lakes, the following may be given :—Measure as many lines round the boundary to be surveyed as will surround it, and tie all the angles, except the two last, as in the preceding examples. There will thus be obtained a system of lines that will prove among themselves, as the last measured side will just reach from the last station to the first, if the work has been done with accuracy.

The outlines of woods, &c., are so various, however, that it would not be advisable in every case to adhere to this method; much must, therefore, be left to the skill of the surveyor.

In the southern counties of England, for instance, where coppice wood is sold by the acre for fuel, it is very frequently required to survey a portion of a wood (the coppice being cut down, and the large timber still standing). In such cases, the lines must be taken within the space to be surveyed (as the adjoining uncut coppice prevents their being taken outside), and be ranged among the growing timber as well as they can, and the tie-lines taken through the most convenient openings left by the trees.

Surveys of this kind are called *traverse* surveys, and (it may be proper to add) are best performed with the help of the prismatic compass, or the box sextant, which—these surveys being always of small extent—are sufficiently accurate for the purpose.

Survey of the Grange Estate.—The plan (Plate I.) and a copy of the field book (Plate II.) relating to this estate are given in the appended plates.

It will be seen that the three angles of each of the triangles A C B, G F E, are given. This is not strictly necessary, as the sides of the triangles were all measured, but the angles were taken by the author when surveying this estate for the purposes of the preliminary work required for the examination of the Institution of Surveyors. Some of the minor filling-in lines have been omitted, and it will be noticed that in the plan all the fences appear as thorn fences, although some were rail fences and are shown as such in the field book. This has been done to render the plan more simple.

Generally, it will be seen that the survey consists of the skeleton triangles A C B and G F E. The first

named triangle is checked by line (4) and is also checked by part of line (8), the second by lines (7) and (8). The triangle G F E will be seen to be well conditioned, whilst the figure of the other is not as uniform.

Beginning at point A, line (1) was first carefully measured to B, pegs being left at all stations. Lines (2) and (3) and the check line (4) were then done, and this work plotted in the office before proceeding further. The lines (5) (6) (7) and (8) were then measured, and plotted as before. Their measurements being found to be correct, the filling-in lines were measured. The angles were taken at the completion of the linear measurements. The copy of the field book will render the system of taking and booking measurements easily intelligible.

Note.—The method described in the text of continuing the measurement of a line impeded by an object, such as a building, is only intended to apply in cases where the line is required but for a short distance beyond the obstruction. In actual practice small errors in setting out the right angles and in measuring the distances, together with small errors in sighting the various poles, may so affect the alignment beyond the impeded point as to render the whole line useless.

CHAPTER V.

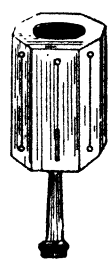
INSTRUMENTS FOR ANGULAR MEASUREMENT.

Magnetic Variation of Declination of the Compass.—The angles with the prismatic compass, and also in one method of using the theodolite, are, as will be shown hereafter, observed with reference to the magnetic meridian, as indicated by the needle of the compass.

The needle does not point to the true meridian or geographical North, owing to terrestrial magnetism, but forms an angle with it, known as the magnetic declination, or variation of the compass.

This angle varies according to locality, as well as daily and yearly. In 1580 the angle was 11° 15′ E., in 1657 0°, in 1818 it was 24° 38′ W., and at the present time (December, 1897) the mean magnetic variation is, as supplied by the courtesy of the Astronomer Royal, 16° 44′ W.

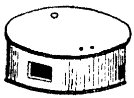
Since this variation will affect equally, or nearly so, all azimuths observed within a limited extent and during a limited time, the angles subtended by any two of the objects observed, being the difference of their azimuths, will not be affected by the variation; and hence the map or plan may be constructed with all the objects in their proper relative positions, but the true meridian must first be laid down on the map, if required, by making allowance for the variation.



The Cross-staff.—The cross-staff can be had in more than one form, but it is commonly made of brass, octagonal in section, with a socket for fastening on to a staff to fix in the ground. Six of the faces have a sight-slit sawn in them, and two have an opening with a vertical hair. The illustration will make the instrument easily intelligible.

This instrument is used (though only rarely) for setting out right angles on the ground, and it will be seen that angles of 45° can also be set out with it. It is at the best unreliable, owing to the line of sight available being short. Its use has now been practically superseded by the optical square described below.

The Optical Square.—For the purpose of measuring long perpendiculars this instrument is now very generally used. It consists of a small circular box containing two mirrors fixed permanently at an angle of 45° to each other, so that any two objects seen in it, the one by direct vision and the other by reflection, subtend at the place of the observer an angle of 90°.

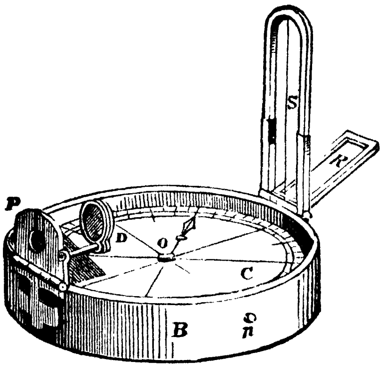


To ascertain the point on the chain-line, at which a line from an object right or left will be perpendicular to it, the observer looking through the instrument in a direct line towards the pole or station in front of him, walks along the chain until the image of the pole or flag set up at the object from which the perpendicular is desired appears to be coincident with the forward object. This is the point at which a line from the object will be perpendicular to the chain-line. If it is desired to set up a perpendicular from a given point on the chain-line, the observer, looking towards the forward station, as before, directs his assistant right or left until the pole of the assistant appears coincident with the forward station.

The Prismatic Compass.—With this instrument horizontal angles can be observed with great rapidity, and, when used with a tripod stand, with a considerable degree of accuracy; it is, therefore, a useful instrument for filling in the details of an extensive survey, after the principal points have been laid down by means of observations, made with the theodolite, hereafter to be described. It was used for this purpose by the Ordnance surveyors.

C is a compass card, divided usually to every 20′, or third-part of a degree, and having attached to its under side a magnetic needle; *n* is a spring, which, being touched by the finger, acts upon the card and checks its vibrations, so as to bring it sooner to rest, when making an observation. S is the sight-vane, having a fine thread stretched along its

opening, which is to cut the point to be observed by the instrument. The sight-vane is mounted upon a hinge-joint, so that it can be turned down flat in the box when not in use. P is the prism, attached to a plate sliding in a socket, and thus admitting of being raised or lowered at pleasure, and also supplied with a hinge-joint, so that it can also be turned down into the box when not in use. In the plate to which the prism is attached, and which projects beyond the prism, is a narrow slit, forming the sight through which the vision is directed when making an observation. On looking through the slit, and raising or lowering the prism in its socket, distinct vision of the divisions on the compass card, immediately under the sight-vane, is soon



obtained; and these divisions, seen through the prism, all appear, as each is successively brought into coincidence with the thread of the sight-vane by turning the instrument round, as continuations of the thread, which is seen distinctly through the part of the slit that projects beyond the prism.

The method of using the instrument is as follows:—the sight-vane S, and the prism P, being turned up on their hinge-joints, as represented in the figure, hold the instrument as nearly in a horizontal position as you can judge, or, if a tripod stand be used, set it as nearly as you can in a horizontal position by moving the legs of the stand, that thence the card may play freely. Raise the prism in its socket till the divisions on the card are seen distinctly

through it, and, turning the instrument round, until the object to be observed is seen through the portion of the slit projecting beyond the prism, in exact coincidence with the thread of the sight-vane, bring the card to rest by touching the spring n; and then the reading at the division upon the card, which appears in coincidence with the prolongation of the thread, gives the magnetic azimuth or bearing of the object observed, or the angle which a straight line, drawn from the eye to the object, makes with the magnetic meridian. The magnetic azimuth of a second object being obtained in the same manner, the difference between these two azimuths is the angle subtended by the objects at the place of the eye, and is quite independent of the error in the azimuths, arising from the slit in the prism not being diametrically opposite to the thread of the sight-vane. As the reading is made on the side of the card nearest the prism, the figures on the card are engraved from right to left.

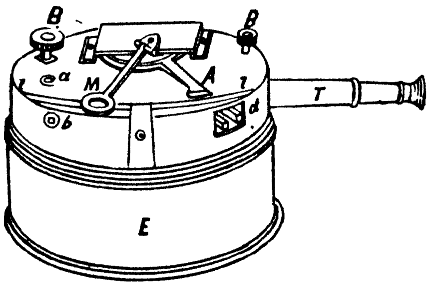
For the purpose of taking the bearings of objects much above or below the level of the observer, a mirror R is supplied with the instrument, which slides on and off the sight-vane S, with sufficient friction to remain at any part of the vane that may be desired. It can be put with its face either upwards or downwards, so as to reflect the images of objects above or below the horizontal plane of the eye of the observer. If the instrument be used for obtaining the magnetic azimuth of the sun, the dark glasses D must be interposed between the sun's image and the eye.

There is also an arrangement not shown in the figure, by touching which a little lever is raised and the card thrown off its centre; as it always should be, when not in use, as the constant playing of the needle would wear the fine steel point, on which it is balanced, and the sensibility of the instrument would be thereby impaired. The sight-vane and prism being turned down, a cover fits on the box, which is about three inches diameter, and one deep; and the whole being packed in a leather-case, may be carried in the pocket without inconvenience.

The Box Sextant.—This instrument, which is equally portable with the prismatic compass—forming, when shut up, a box about three inches in diameter, and an inch and a half deep—will measure the actual angle between any two

objects to a single minute. It requires no support but the hand, is easily adjusted, and, when once adjusted, seldom requires re-adjusting.

When the sextant is to be used, the lid E of the box is taken off and screwed to the bottom, where it makes a convenient handle for holding the instrument; the telescope, T, being then drawn out, the instrument appears as shown in the figure. A is an index arm, having at its extremity a vernier, of which 30 divisions coincide with 29 divisions on the graduated limb *l l*, and the divided spaces on the limb denote each 30 minutes, or half a degree, the angles observed being read off by means of the vernier to a single minute. (The principle of the vernier is described at the end of this chapter.) The index is moved by turning the milled head B, which acts upon a rack and pinion within



the box. To the index arm is attached a mirror, called the index glass, which moves with the index arm, and is firmly fixed upon it by the maker, so as to have its plane accurately perpendicular to the plane in which the motion of the index arm takes place, called the plane of the instrument; this plane is evidently the same as the plane of the face of the instrument, or of the graduated limb *l l*. In the line of sight of the telescope is placed a second glass, called the horizon glass, having half its surface silvered, and which must be adjusted that its plane may be perpendicular to the plane of the instrument, and parallel to the plane of the index glass, when the index is at zero. The instrument is provided with two dark glasses, which can be raised or lowered by the little levers seen at *d*, so as to be interposed, when necessary, between the mirrors and any object too

bright to be otherwise conveniently observed, as objects in the direction of the sun. The eye-end of the telescope is also furnished with a dark glass, to be used when necessary.

To see if the instrument be in perfect adjustment.—Place the dark glass before the eye-end of the telescope, and looking at the sun, and moving the index backwards and forwards a little distance on either side of zero, the sun's reflected image will be seen to pass over the disc, as seen directly through the horizon glass, and if in its passage the reflected image completely covers the direct image, so that one perfect orb is seen, the horizon glass is perpendicular to the plane of the instrument: but, if not, the screw at *a* must be turned by the key *k* till such is the case. The key *k* fits the square heads of both the screws seen at *a* and *b*, and fits into a spare part of the face of the instrument, so as to be at hand when wanted. This adjustment being perfected, bring the reflected image of the sun's lower limb in exact contact with the direct image of his upper limb, and note the reading of the vernier; then move the index back beyond the zero division of the limb, till the reflected image of the sun's upper limb is in exact contact with the direct image of his lower limb, and, if the zero of the vernier be now exactly as far behind the zero of the limb, as it was at the former reading in front of it, the instrument is in perfect adjustment; but, if not, half the difference is the amount of error, which must be corrected by applying the key *k* to the screw at *b*, and turning it gently till both readings are alike, each being made equal to half the sum of the two readings first obtained. When this adjustment is perfected, if the zeroes of the vernier and limb are also made exactly to coincide, the reflected and direct image of the sun will exactly coincide, so as to form but one perfect orb, and the reflected and direct image of any line, sufficiently distant not to be affected by parallax, as the distant horizon, or the top or end of a wall, more than half a mile distant, will coincide so as to form one unbroken line.

To obtain the angle subtended by two objects in, or nearly in, the same horizontal plane.—Hold the sextant in the left hand, and bring the reflected image of the right-hand object into coincidence with the direct image of the left-hand object, and the reading of the instrument will give the angle between the two objects.

To obtain the angle subtended by two objects in, or nearly in, the same vertical plane.—Hold the instrument in the right hand, and bring down the reflected image of the upper object by turning the milled head B, till it exactly coincides with the direct images of the lower object, and the reading of the instrument will give the angle between the two objects.

It will be seldom that the surveyor need pay any attention to the small error arising from parallax, but, should great accuracy be desirable, and one of the objects be distant while the other is near, the parallax will be eliminated by observing the distant object by reflection, and the near one by direct vision, holding the instrument for this purpose with its face downwards, if the distant object be on the left hand. If both objects be near, the reflected image of a distant object, in a direct line with one of the objects, must be brought into coincidence with the direct image of the other object, and the parallax will thus be eliminated.

For the purposes of surveying, the horizontal angles between objects are chiefly required, and the reduction of these angles from the actual oblique angles subtended by the objects, would be a troublesome process. If the angle subtended by two objects be large, and one be not much higher than the other, the actual angle observed will, however, be a sufficiently near approximation to the horizontal angle required ; and if the angle between the two objects be small, the horizontal angle may be obtained, with sufficient accuracy, by taking the difference of the angles observed between each of the objects, and a third object at a considerable angular distance from them. With a little practice the eye will be able to select an object in the same direction as one of the objects, and nearly on a level with the other object, and the angle between this object and the object selected will be the horizontal angle required.

For laying off long offsets, or perpendicular distances from a station line.—The pocket sextant is a most convenient instrument for this purpose : for by setting the index to 90°, and walking along the station line, looking through the horizon glass directly at the further station staff, or any other remarkable object on the station line, any object off the station line will be seen by reflection, when the observer arrives at the point where the perpendicular falls from this object upon the station line, and the distance from this

point to the object, being measured, is its perpendicular distance from the station line.

The Theodolite.—The theodolite is the most important angular instrument used by surveyors, and measures both the horizontal angles between two objects and the angles of elevation or depression formed by these objects with the point of observation.

There are several forms of theodolite manufactured, differing in detail, the main parts being common to all. The theodolite here described is that most generally in use, and is known as the "transit" theodolite, from the fact of the telescope being fitted with a complete vertical circle, mounted in the same way as the transit instrument used in astronomical observations.

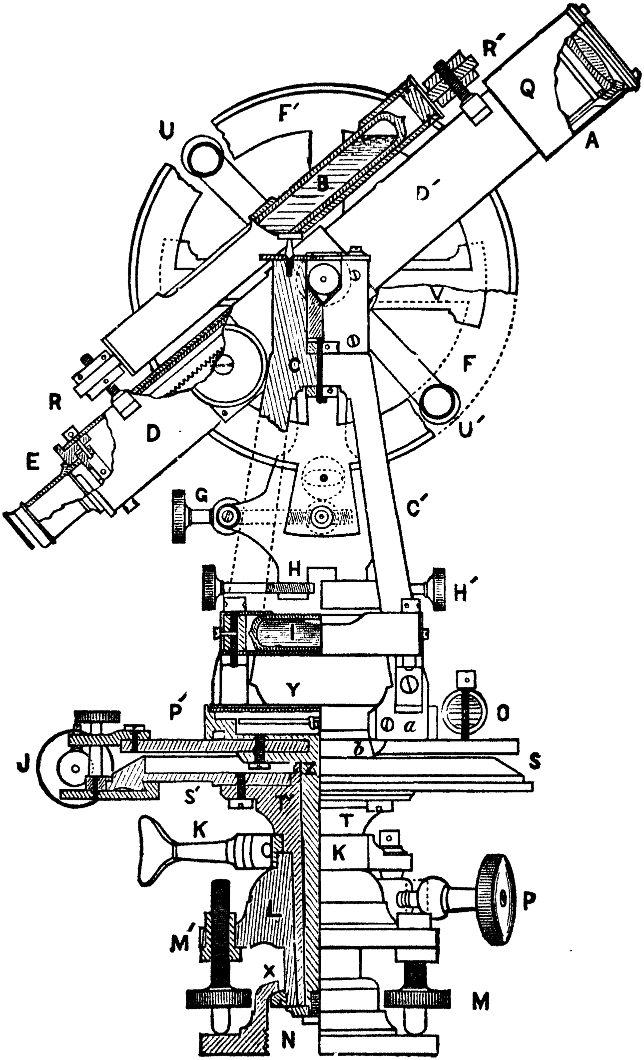
By the courtesy of Mr. W. F. Stanley, of Great Turnstile, London, the drawings showing the construction of the instrument are here produced, and the following description is mainly compiled from his work on "Surveying and Levelling Instruments." The theodolite comprises—

A tripod stand of mahogany fitted with screw for fixing to the lower parallel plate. A metal stand is supplied also with the instrument, for using instead of the tripod, when it is desired to place the instrument on a wall for taking observations.

The lower parallel plate N has a large boss piece taking up from its central part which forms a hollow dome called the socket shown at X. The interior of the lower part of N is cut into a coarse female screw for attaching the instrument to the tripod.

The upper parallel plate is constructed as a flange from a solid boss L. The boss is prolonged downwards by a stem piece upon the lowest part of which a ball collar of globular section is firmly screwed. The ball collar fits into the socket carried up from the lower parallel plate The whole of this globular arrangement is called the ball and socket. The boss L of the upper parallel plate with its stem has a hollow conical hole through its axis into which the body piece to be described fits accurately. Upon its outer upper part a collar is formed which acts as a guide to the clamp K. The plate is tapped in four places for parallel screws.

The parallel plate screws. One is shown in elevation at M with its top dotted and one in section at M′. The four


R'
Q
A
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J
S
S'
T'
T
K
K
P
L
M'
X
M
N

parallel plate screws are in opposite pairs placed exactly at right angles to each other in a line passing through the vertical axis of the instrument. In some instruments an arrangement of three screws, instead of the four here mentioned, is used for levelling the horizontal limb.

The body piece.—This is shown in elevation at T and in section at T′. The limb of the instrument S S′ is attached to it by screws. The clamp K K bites upon it. The lower outer part of the body piece forms a conical fitting in the boss of the upper parallel plate L. The interior is a hollow conical axis.

Axis collar clamp K.—The clamp surrounds the body piece and clamps it by means of screw K shown on the left hand. The clamp is connected with the upper parallel plate through the tangent screw the head of which is shown at P, so that when the screw K is tightened the parts L and T are fixed together, except that a slow motion can be given to these parts by the tangent screw P.

By this clamp and tangent arrangement the whole of the upper part of the instrument is rendered free to revolve when the clamp is loosened to bring the instrument to bearing, the final adjustment being secured after clamping, by the tangent screw.

The central vertical axis is shown in half section at Z and is madé in the form of a truncated cone; its fitting surfaces are at the two ends of the cone extending about half an inch, the central part being chambered back.

At its upper part it is screwed by a wide collar to the vernier plate. A square-hole collar and screw is fixed on the lower end of the axis just to touch the socket of the body piece so as to secure the axis in position when the instrument is lifted. An eye or hook (not shown in the drawing) is fixed into the screw at the lower end to take the cord of the plummet used for fixing the instrument over a definite point in the ground.

The horizontal or lower plate or limb.—The whole of the piece S S′ is called the limb, but more generally only the divided part. It is attached to the body piece by screws. The outer rim is divided to 30′ in angles from 0° to 360° reading from left to right. It also supports the clamp piece.

The vernier plate is shown in section under P′ and is carried from the central axis Z. It carries the ball nut of

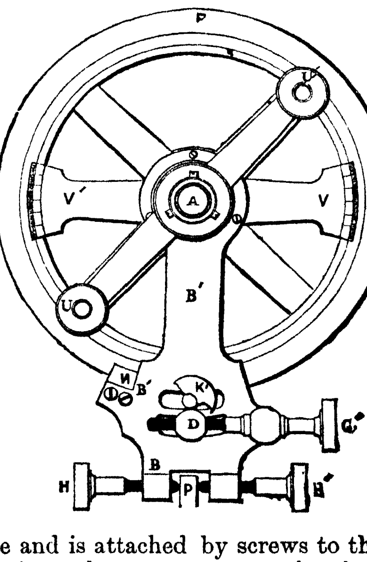
the tangent screw. The verniers are read to minutes by a pair of microscopes so placed that when the vernier is being read the other on the opposite side can be read also. The vernier plate also carries a spirit-level at O. (The principle of the vernier is described at the end of this chapter.)

The compass box is fixed to vernier plate and the needle reads into a divided circle of 360°. In some instruments a trough compass with a long needle is used.

The A frames, shown in C C′, are attached to the vernier plate. Upon the front of one of them a spirit-level I is placed at right angles to that in the vernier plate at O. On the inside of each standard is a clip piece for taking the clipping screws to be described. The transit axis rests on two V's at the top of the standard.

The transit axis fits into the V's and at its centre is formed into a collar, clipping the outer tube of the telescope. The vertical circle is attached to the collar by means of a flange.

The vertical vernier frame V V is centred up in front of the



vertical circle and is attached by screws to the "clips." It has two verniers, the one at an angle of 180° from the other. Two microscopes are provided for reading.

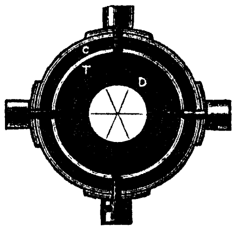
The clips.—The clipping arm which is centred on the transit axis is shown B B′ B′. It is attached to the verniers, but moves freely on its axis at A so as to permit unrestrained motion of the telescope. A clamp working in a slot is shown at K′. It is used for fixing the vernier stationary, except for adjustment by the tangent screw G′. This clamp and tangent set the vernier to zero. It is also used in setting the telescope before angles of altitude or depression are measured. The clipping screws H H′ bring the principle bubble B to the centre of the run after the vernier has been brought to zero by means of the clamp and tangent screws. The clipping screws hold either of the clips to one standard or the other.

The vertical adjustment is equivalent to the horizontal already described, but in vertical plane.

The vertical circle is carried by four arms from a central boss attached by screws to the transit axis. The circle is divided generally to half degrees, and is figured to right and left from zero both ends. The zero lines are directly coincident with the optical axis of the instrument. The vernier reads to minutes in either direction, the rising arc above the level datum being considered as plus, the falling arc as minus.

The telescope consists of a pair of triblet-drawn tubes, with the object-end enlarged to take the object glass ; a ray shade fits over the object-end. The inner tube of the telescope slides forward to or from the objective by means of a rack and pinion, moved by a large milled-headed screw. The eye-piece is known as the Ramsden eye-piece, and is termed a "positive" eye-piece. It consists of two plano-convex lenses, the convex-cut faces of which are turned towards each other. It slides in a tube at the end of the telescope to focus upon the diaphragm. It is also sometimes called an "inverting" eye-piece, but is not really so, as it is the object glass that inverts the image of the object viewed, and the eye-piece picks up the image in its inverted position. An erecting eye-piece is also supplied with glasses so arranged that the image brought to focus inverted is again erected so that objects appear in their natural position. This, however, entails loss of light, so that to obtain an equal clearness of vision with an erecting eye-piece, as with a Ramsden eye-piece, a correspondingly larger object glass must be used.

The diaphragm of the telescope is so constructed as to permit displacement of spiders' webs in any direction at right angles to the axis of the telescope, the object being to adjust the crossing of the webs to the axis of the telescope.



The diaphragm is a stout disc of brass held in its place and adjusted by four capstan-headed screws termed collimating screws. The diaphragm has generally three spider webs crossed as shown, but is sometimes made of glass with the lines finely marked on.

Adjustments of the Theodolite.—The following six adjustments are required. Of these Nos. 1, 2, and 6 must be made on each occasion that the instrument is used; the remainder, viz., Nos. 3, 4, and 5, are made by the maker before the instrument leaves the works, and with ordinary care will not require re-adjusting except at considerable intervals. They should, however, be examined from time to time to see if they are accurate.

1. *Setting up the instrument in the centre of the exact point of observation.*—As previously stated, a pointed plummet and cord are supplied with the instrument, and a hook is attached to the central vertical axis for hanging the cord to. The instrument should be set up approximately at first, as level as possible by the eye, the lower parallel plate being observed for this purpose. The legs are then moved until the centre of the plummet touches the exact centre of the peg over which the instrument is placed.

2. *Adjustment for parallax.*—Parallax is a term in general use to denote the difference between the true and apparent place of any object, or in other words, when the image of the object viewed, formed by the object glass, falls either short of or beyond the place of the cross lines, the error arising from this cause is called parallax.

Adjustment.—Draw out the tube of the eye-piece until the cross hairs appear to be defined and to be free from effects of parallax that may arise from optical displacement of cross hairs when the eye is placed in a lateral position. If, when the telescope of the instrument is pointed to any distant object, its image remains fixed, when the eye is

moved to the right or the left, or in a lateral position, out of the optical axis of the telescope, no parallax exists; on the contrary, if the image of the distant object does not remain steady the tube must be drawn out, more or less, until the required steadiness of the image takes place. As an additional precaution against errors arising from parallax, observations should always be taken as nearly as possible through the centre of the eye-glass.

3. *Testing the line of collimation.*—The line of sight passing through the cross hairs of the diaphragm, the optical axis of the instrument and the object sighted, is the line of collimation. The adjustment of this consists in placing it coincident with, or failing that, parallel to, the mechanical axis of the telescope, by manipulating the collimating screws of the diaphragm. To ascertain whether there is an error of collimation, direct the telescope to some well-defined object at a distance, and see that the intersection of the cross lines cuts it accurately. Unclamp the vertical circle and transit the telescope without disturbing it in azimuth. Have a mark fixed exactly at the intersection of the webs on a wall or other convenient place, and read the vertical circle. Turn the instrument through 180° in azimuth, and again sight the back station. Transit the telescope as before, and sight on to the mark previously made. The telescope is now inverted, and if the collimation is correct the webs will still come on the mark, and the vertical circle will read the same as before. If it is found, upon repeating this procedure, that the webs still cannot be made to intersect the mark when the telescope is inverted, the diaphragm requires adjusting. See Appendix IV.

4. *Adjustment of the horizontal limb.*—Clamp the vertical axis by means of the screw K, unclamp the vernier plate, fix the vertical arc at or near zero, turn it round till the telescope is directly over two of the parallel plate screws. Bring the bubble of the level of the telescope to the centre of its run by turning the tangent screw. Turn the vernier plate half round, bringing the telescope again over the same pair of parallel plate screws; and, if the bubble of the level be not still in the centre of its run, bring it back to the centre, half way, by turning the parallel plate screws, over which it is placed, and half way by turning the tangent screw. Repeat this operation till the bubble

remains accurately in the centre of its run, in both positions of the telescope; and, then turning the vernier plate round till the telescope is over the other pair of parallel plate screws, bring the bubble again to the centre of its run by these screws. The bubble will now retain its position while the vernier plate is turned completely round, showing that the internal axis about which it turns is completely vertical. The bubbles of the levels on the vernier plate must now be adjusted by the capstan screws, and then show the axis to be vertical. Now, having clamped the vernier plate, unclamp the screw K, and move the instrument slowly round on the external axis, and, if the bubble of the level maintain its position during a complete revolution, the external and internal axes are coincident, both being vertical at the same time; but, if the bubble does not maintain its position, it shows that two parts of the axis have been inaccurately fitted, and the fault can only be remedied by the instrument-maker.

5. *Adjustment of the vertical limb.*—The bubble of the main level being in the centre of its tube, as described in the last operation, read the two verniers to the vertical circle. If these read zero the instrument is probably in adjustment. If any deviation from zero is observed, bring the verniers exactly on to it by means of the tangent screw. This will throw the bubble on the telescope out of centre, so it must be brought back by means of the clipping screws H, H′ in the diagrams (pp. 61 and 63). Now, provided that the line of collimation in altitude is correctly adjusted, also that the bubble tube is exactly parallel to the line of collimation, and the verniers to the vertical circle have been properly set, the instrument will be in adjustment. It is sometimes found, however, that a small error exists through the latter condition not being exactly complied with. When this occurs it cannot be eliminated by the adjustments so far described, but its exact amount can be ascertained and allowed for as an index error by applying it + or − to each vertical angle observed. For further information on this subject, see Appendix IV.

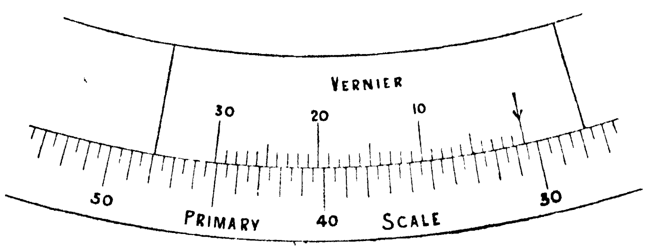
This deviation is best determined by repeating (as is explained in the chapter on Surveying with the Theodolite) the observation of an altitude or depression in the reversed positions both of the telescope and vernier plate. The two readings will have equal and oppo-

site errors, and the half of the difference will be the index error.

6. *Levelling the parallel plates.*—This is effected by the screws M M′ and the two levels O and I attached to the vernier plate. The vertical axis is clamped by the screw K, and the instrument is placed so that the telescope is directly over two of the screws.

The two levels O and I are now each parallel to two of the parallel plate screws. The vernier plate is then clamped. Should the bubble of the tube I not be in the centre of the run, but towards the right-hand extremity of the tube, the parallel plate screws M M′ parallel with the level I must both be turned outwards, thus depressing the right-hand corner of the plate from the vertical axis of the instrument till the bubble is in the centre. Had the bubble been at the left extremity of the tube, the screws parallel with the level I should have been turned inwards towards the axis of the instrument. When the bubble of the level I is duly level, the telescope should be turned parallel to the level O, and the same steps should be taken in the case of this bubble tube, and the screws parallel with it. It is customary to level the tubes I and O a second time each, as the adjustment of the one sometimes upsets that of the other.

The Vernier Scale.—The vernier is so named from its inventor, Peter Vernier. It is used in conjunction with,



and forms subdivisions to, another scale called the primary scale, which, if divided to the same extent as the vernier, would have its parts so fine as to be indistinct.

On the theodolite the primary scale is generally only divided into degrees and half degrees, or 30 minutes. The

vernier enables us to read the additional number of minutes in the angle observed.

The principle is thus stated :—If a space on the primary scale be divided into a given number of parts equal to $n-1$, and a space equal in length to the first be divided on the vernier or moveable scale into a number of parts equal to n, these parts will each be smaller than the first by the nth part of a division on the primary scale. If one inch be divided on the primary scale into 10 equal parts and a space on the vernier equal to 9 of the parts be itself divided into 10 equal parts, the difference between a division on the primary scale and a division on the vernier will equal to $\frac{1}{10}$ of the first and therefore to $\frac{1}{100}$ of an inch.

Similarly, on a theodolite divided on the primary scale into divisions representing 30 minutes, a space is set out on the vernier equal to 29 of these divisions, and this space so set out is divided into 30 parts, each division on the vernier differing from that on the primary scale by $\frac{1}{30}$, or 1 minute. To read the instrument an account is first taken of the degrees and half degrees (if any) to which the index of the vernier (marked thus on it $\downarrow$) points, and the eye is passed along the vernier until some one of its lines coinciding with any line on the primary scale is found.

The number of divisions on the vernier or minutes between the index of the vernier and such coinciding line with the primary scale is then added to the first account taken of degrees or half degrees. In the illustration the angle is 30°, 37′.

The Plane Table is another instrument largely used for surveying purposes in countries where the weather can be relied upon. Its extreme simplicity is a great recommendation in some cases, as it can be used by a practically unskilled assistant. For the benefit of those who might be called upon to use this instrument a description is given on page 238.

CHAPTER VI.

LOGARITHMS AND TRIGONOMETRY AS APPLIED IN SURVEYING.

LOGARITHMS.

BY **Logarithms**, calculations are rendered more easy, for by a table of logarithms multiplication is changed into addition, division into subtraction, involution into multipli cation, and evolution into division.

The logarithm of a number to a given base is the index of the power to which the base must be raised to give that number. The base of the logarithms registered in the ordinary tables is 10. Hence the logarithm of 10 is 1, for $10 = 10^1$ and the logarithm of 100 is 2, since $100 = 10^2$.

Logarithms are calculated to 7 places of decimals, and the integral parts of the logarithms of numbers higher than 10 are called the characteristics, and the decimal parts of the logarithm the mantissæ.

For example, the logarithm of 80 is 1·9030900, where 1 is the characteristic and ·9030900 the mantissa. The characteristic of the logarithm of a number is always one less than the number of integral figures of a number. Hence, if the number contain 4 integral figures, the characteristic is 3, if it contain 3 integral figures the characteristic is 2, and so on.

Various published tables give the mantissæ of the logarithms of numbers up to 10,000, and from these the characteristic may be fixed in accordance with the rule above stated. For example :—

Log. of 8743 is 3·9416605
Log. of 874·3 is 2·9416605
Log. of 87·43 is 1·9416605
Log. of 8·743 is 0·9416605

If the number has no integral figures the characteristics are always negative, the mantissæ remaining positive. If there is no cipher after the decimal of the number the characteristic is — 1 ; if there is one cipher the characteristic is — 2, and so on. In calculations with negative characteristics the rules of algebra are followed. The same mantissæ serve for the logarithms of all numbers, whether greater or less than unity, which have the same significant digits. For example :—

$$\text{Log. of } \cdot 8743 \text{ is } \bar{1}\cdot 9416605$$
$$\text{Log. of } \cdot 08743 \text{ is } \bar{2}\cdot 9416605$$

The reader is referred for finding, from the tables of logarithms, the logarithm of a number, or for finding the number corresponding to a given logarithm, to the explanation published with such tables.

Multiplication by Logarithms.

RULE.—Add the logarithm of the number, and the sum will be the logarithm of the product.

Example.—Multiply 231·4 and 5·062.

$$\begin{array}{rl} \text{Log. } 231\cdot 4 = & 2\cdot 3643634 \\ \text{Log. } 5\cdot 062 = & \underline{0\cdot 7043221} \\ & 3\cdot 0686855 \end{array}$$

or logarithm of 1171·347, which is the required product.

Division by Logarithms.

RULE.—From the logarithm of the dividend, subtract that of the divisor, and the remainder will be the logarithm of the quotient.

Example.—Divide 241·63 by 4·567.

$$\begin{array}{rl} \text{Log. } 241\cdot 63 = & 2\cdot 3831509 \\ \text{Log. } 4\cdot 567 = & \underline{0\cdot 6596310} \\ & 1\cdot 7235199 \end{array}$$

and 1·7235199 = logarithm of 52·90782, which is the required quotient.

Involution by Logarithms.

RULE.—Multiply the logarithm of the given number by the exponent of the power to which it is to be raised, and the product will be the logarithm of the required power.

Example.—Find the fourth power of 9·163.

$$\text{Log. } 9{\cdot}163 = 0{\cdot}9620377$$
$$\times 4$$
$$3{\cdot}8481508$$

and 3·8481508 = logarithm of 7049·38, which is the required product.

Evolution by Logarithms.

RULE.—Divide the logarithm of the given number by the exponent of the root which is to be extracted, and the quotient will be the logarithm of the required root.

Example.—Find the cube root of 12345.

$$\text{Log. } 12345 = 4{\cdot}0914911$$

Dividing by 3 we get 1·3638304 = logarithm of 23·11162,

$$\text{or } \sqrt[3]{12345} = 23{\cdot}11162.$$

Tables of Natural Sines and Cosines, etc., of Logarithmic Sines, Cosines, etc.

Trigonometrical ratios, as, for example, sine, cosine, &c., are, as is shown later, numerical quantities.

There are two forms of tables :—

(1) The table of natural sines, cosines, &c.

(2) The table of logarithmic sines, cosines, &c.

The logarithmic sines, cosines, &c., are called tabular logarithms, and are denoted by the letter L prefixed.

The natural sines and cosines can be ascertained from the logarithmic sines and cosines by subtracting 10 from the indices of the latter, the number corresponding to this logarithm being the natural sine or cosine. The number 10 was added to prevent the frequent occurrence of negative quantities in tables.

Example.—L sin. A = log. sin A + 10.

TRIGONOMETRY.

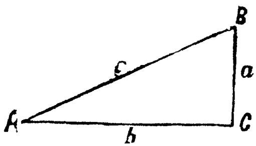
Consideration of Spherical Excess.—Owing to the figure of the earth being that of an oblate spheroid, in any triangulation taken in a survey the triangles are spheroidal, the curvature of the sides having to be taken into consideration ; but it has been proved that, without appreciable

error, a spheroidal triangle can be estimated as spherical. Now the angles in a spherical triangle exceed two right angles, the excess varying according to the area of the triangle, the radius of the sphere, and other considerations presenting great difficulty in the calculations.

In ordinary operations, however, the spherical excess does not exceed four seconds. The solution is given by Legendre in the following theorem, "that, in any spherical triangle, the sides of which are small proportionately to the radius of the sphere, if each of the angles be diminished by one-third of the spherical excess (so as to reduce their sum to two right angles) these diminished angles may be treated as those of a plane triangle, having sides equal in length to the length of the spherical arc sides straightened out into rectilinear lines." Thus the parts of a spherical triangle can be worked out as if the triangle were a plane triangle.

Trigonometrical Ratios.

Let A B C be a right-angled triangle. If the angles are denoted by A, B, and C, and the sides by a, b and c, we shall have the following ratios :—



Sine of angle A (written sin. A) $= \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{a}{c}$

Cosine ,, (,, cos. A) $= \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{b}{c}$

Tangent ,, (,, tan. A) $= \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{a}{b}$

Cosecant ,, (,, cosec. A) $= \dfrac{\text{hypotenuse}}{\text{perpendicular}} = \dfrac{c}{a}$

Secant ,, (,, sec. A) $= \dfrac{\text{hypotenuse}}{\text{base}} = \dfrac{c}{b}$

Cotangent ,, (,, cot. A) $= \dfrac{\text{base}}{\text{perpendicular}} = \dfrac{b}{a}$

Concise Formulæ, &c.

Complement of an angle = the excess of a right angle over that angle.

Example.—Complement of angle $A = 90^\circ - A$.

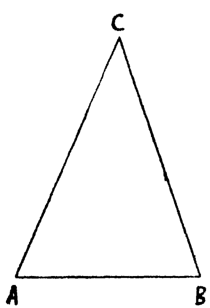
Supplement of an angle = excess of two right angles over that angle.

Example.—Supplement of angle $A = 180^\circ - A$.

$$\begin{aligned}
&\text{Sin. } (90^\circ - A) = \text{cos. } A. \\
&\text{Tan. } (90^\circ - A) = \text{cot. } A. \\
&\text{Sec. } (90^\circ - A) = \text{cosec. } A. \\
&\text{Sin. } (180^\circ - A) = \text{sin. } A. \\
&\text{Cos. } (180^\circ - A) = -\text{cos. } A. \\
&\text{Tan. } (180^\circ - A) = -\text{tan. } A. \\
&\text{Cot. } (180^\circ - A) = -\text{cot. } A. \\
&\text{Sec. } (180^\circ - A) = -\text{sec. } A. \\
&\text{Cosec. } (180^\circ - A) = \text{cosec. } A. \\
&\text{Vers } (180 - A) = 1 + \text{cos. } A.
\end{aligned}$$

Algebraical Signs of Ratios in the Respective Quadrants.

Ratio.	First Quadrant.	Second Quadrant.	Third Quadrant.	Fourth Quadrant.
Sin. . . .	+	+	–	–
Cos. . . .	+	–	–	+
Tan. . . .	+	–	+	–



Determination of Inaccessible Heights and Distances.

1. *To determine the distance of an inaccessible point on a horizontal plane.*

Let C = inaccessible point.

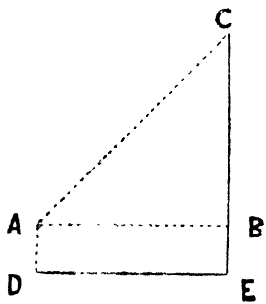
RULE.—Measure any length A B. Observe angles C A B and C B A,

$$\text{then } A\,C = \frac{A\,B \text{ sin. } A\,B\,C}{\text{sin. } A\,C\,B}$$

2. *To measure height of accessible object.*

Let C E be the accessible object.

RULE.—Measure any distance E D from foot of object, the line A B representing a line parallel with the ground at the height of the observer's eye; from it measure distance between same and ground, and observe the angle C A B,

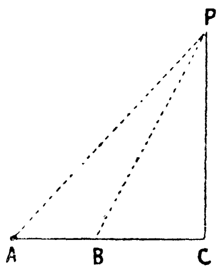


then C E = B E (or A D) + B A tan. C A B.

3. *To measure distance and height of inaccessible object.*

Let P C = inaccessible object.

RULE.—Measure any length A B towards P C. Take angles P A B and P B C. (By Euclid I. 32 A P B = difference between P B C and P A C.)



$$\text{then } P\,C = \frac{A\,B \text{ sin. } P\,A\,B \text{ sin. } P\,B\,C}{\text{sin. } A\,P\,B}$$

$$\text{and } A\,C = \frac{A\,B \text{ cos. } P\,A\,B \text{ sin. } P\,B\,C}{\text{sin. } A\,P\,B}$$

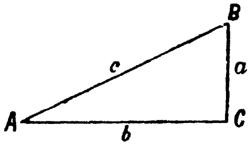
4. The following proposition is of the utmost importance in trigonometrical calculation, viz., *In any triangle the sides are proportional to the sines of the opposite angles.*

Solution of Triangles.

A triangle consists of six elements—viz., three sides and three angles—and a certain number being given, by calculation the remaining unknown elements can be ascertained. When the three angles only are given, the ratio only can be ascertained that the sides bear to one another, and not their actual lengths.

Solution of Right-Angled Triangles.

Let A B C be a right-angled triangle, and the angles be denoted by A, B and C, and the sides a, b and c, as in the figure :—



Given.	Sought.	Formulæ.
a, c	A, B, b	$\sin. A = \frac{a}{c}$, $\cos. B = \frac{a}{c}$, $b = \sqrt{(c+a)(c-a)}$, or $\sqrt{c^2 - a^2}$
a, b	A, B, c	$\tan. A = \frac{a}{b}$, $\cot. B = \frac{a}{b}$, $c = \sqrt{a^2 + b^2}$
A, a	B, b, c	$B = 90^\circ - A$, $b = a \cot. A$, $c = \frac{a}{\sin. A}$
A, b	B, a, c	$B = 90^\circ - A$, $a = b \tan. A$, $c = \frac{b}{\cos. A}$
A, c	B, a, b	$B = 90^\circ - A$, $a = c \sin. A$, $b = c \cos. A$

Example.—Given the side $a = 124{\cdot}6$ yds.
$A = 64^\circ\ 20'$
$C = 90^\circ$

required angle B and sides b and c.

$$B = 180 - (90^\circ + 64^\circ\ 20)$$
$$= 25^\circ\ 40'$$
$$b = a \cot. A$$
$$\log. b = \log. a + L \cot. A$$
$$= \log. 124{\cdot}6 + L \cot. 64^\circ\ 20' - 10$$
$$= 2{\cdot}0955180$$
$$9{\cdot}6817396$$
$$\overline{1{\cdot}7772576}$$

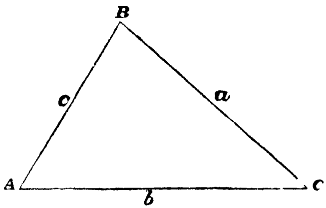
$b = 59{\cdot}876$ yds.

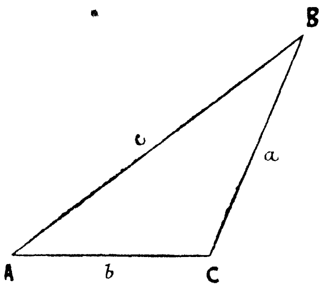
or b can be found by Euc. I., 47. $b = \sqrt{c^2 - a^2}$

$$c = \frac{a}{\sin. A}$$

$$\begin{aligned} \log. c &= \log. a - \text{L} \sin. \text{A} + 10 \\ &= \log 124{\cdot}6 - \text{L} \sin. 64^\circ 20 + 10 \\ &= 2{\cdot}0955180 \\ &\ 10{\cdot} \\ \hline &\ 12{\cdot}0955180 \\ &\ 9{\cdot}9548834 \\ \hline &\ 2{\cdot}1406346 \\ c &= 138{\cdot}24 \text{ yds.} \end{aligned}$$

Solution of Oblique-Angled Triangles.





Given.	Sought.	Formulæ.
A, B, a	b	$b = \dfrac{a \sin. \text{B}}{\sin. \text{A}}$
A, a, b	B	$\sin. \text{B} = \dfrac{b \sin. \text{A}}{a}$
a, b, C	A—B	$\tan. \frac{1}{2} (\text{A}-\text{B}) = \dfrac{(a-b) \tan. \frac{1}{2} (\text{A} + \text{B})}{a + b}$
a, b, c	A	$\sin. \dfrac{\text{A}}{2} = \sqrt{\dfrac{(s - b)(s - c)}{b\,c}}$ where $s = \dfrac{a + b + c}{2}$

It will be seen from the formulæ that the following cases of solving triangles present themselves :—

I. When two angles and a side are given, of which the side may be—

(1) The side between the two angles, or
(2) A side opposite one of the two angles.

II. When an angle and two sides are given, which sides may be—

(1) The sides containing the angle between them, or
(2) Sides not containing the angle between them.

In this last case—known as the "ambiguous" case—let a and b be the given sides, and A the given angle.

Now by the formula $b = \frac{a \text{ sin. B}}{\text{sin. A}}$

$$\therefore \text{sin. B} = \frac{b \text{ sin. A}}{a}$$

and* if $\frac{b \text{ sin. A}}{a}$ is less than unity, two different angles may be found less than 180° which have $\frac{b \text{ sin. A}}{a}$ for sine, one of these angles being less than a right angle and the other greater. If a be not less than b, then A must be not less than B, and therefore B must be an acute angle, and only the smaller value is admissible for B. If a is less than b, then either value might be taken for B. When B is determined, C is known, since it is 180 − A − B, and then c can be found from $\frac{c}{a} = \frac{\text{sin. C}}{\text{sin. A}}$

Thus if two values are admissible for B we obtain two corresponding values for C and c, so that two triangles can be found from the given elements.

III. When the three sides are given.

Examples of the Foregoing Cases.

I. *To solve a triangle, having given two angles and a side.*

$$\begin{aligned} \text{In } \triangle \text{ A B C given } a &= 1000 \text{ yds.} \\ \text{B} &= 104° \\ \text{C} &= 24° \; 29' \end{aligned}$$

$$\text{then A} = 180° - (104° + 24° \; 29') = 51° \; 31'$$

$$\text{now } b = \frac{a \text{ sin. B}}{\text{sin. A}}$$

$$\begin{aligned} \log. b &= \log. a + \text{L sin. B} - \text{L sin. A} \\ &= \log. 1000 + \text{L sin. } 104° - \text{L sin. } 51° \; 31' \end{aligned}$$

* Todhunter's "Trigonometry."

$$\begin{array}{r} 3{\cdot}0000000 \\ 9{\cdot}9869041 \\ \hline 12{\cdot}9869041 \\ 9{\cdot}8936448 \\ \hline 3{\cdot}0932593 \end{array}$$

$$b = 1239{\cdot}54 \text{ yds.}$$

II. (2) *To solve a triangle, having given two sides and the angle opposite to one of them.*

Given $a = 528$ yds., $b = 252$ yds.
$A = 124° 34'$, find B and C.

In this case the ambiguity does not exist as, since A is obtuse, both the angles B and C must be acute.

$$\text{Sin. B} = \frac{b \text{ sin. A}}{a}$$

$$\text{L sin. B} = \log. b + \text{L sin. A} - \log. a$$
$$= \log. 252 + \text{L sin. } 124° 34' - \log. 528.$$

$$\begin{array}{r} 2{\cdot}4014005 \\ 9{\cdot}9156460 \\ \hline 12{\cdot}3170465 \\ 2{\cdot}7226339 \\ \hline 9{\cdot}5944126 \end{array}$$

$$\therefore \text{B} = 23° 8' 33''$$
$$\text{C} = 180° - 124° 34' - 23° 8' 33'' = 32° 17' 27''$$

To solve a triangle, having given two sides and the angle opposite to one of them.

The ambiguous case.

$$a = 397{\cdot}4$$
$$b = 1249{\cdot}7$$
$$\text{A} = 9° 20'$$

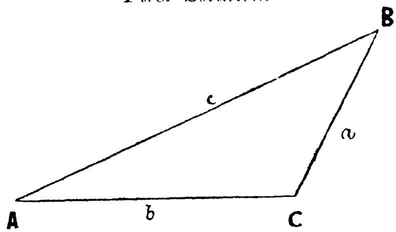
In this case the angle A is acute, and the side b is greater than the side a. B is, therefore, greater than A (Euc. I. 18), but this does not settle the difficulty, because B can be either acute or obtuse for sin. B = sin. (180° − B).

In trigonometrical surveying the ambiguity can be settled

by referring to the field record, as to whether B is acute or obtuse in accordance with the known data.

The example is, however, worked out below, showing the two different solutions.

First Solution.



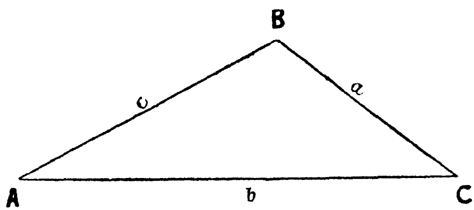
$$\text{Sin. B} = \frac{b \text{ sin. A}}{a}$$

$$\text{L sin. B} = \text{log. } b + \text{L sin. A} - \text{log. } a$$
$$= \text{log. } 1249{\cdot}7 + \text{L sin. } 9^\circ\ 20' - \text{log. } 397{\cdot}4.$$

$$\begin{array}{r} 3{\cdot}0968058 \\ 9{\cdot}2099917 \\ \hline 12{\cdot}3067975 \\ 2{\cdot}5992279 \\ \hline 9{\cdot}7075696 \end{array}$$

then $B = 30^\circ\ 39'\ 50''$ and
$C = 180^\circ - 9^\circ\ 20' - 30^\circ\ 39'\ 50'' = 140^\circ\ 0'\ 10''$
and $c = 1574{\cdot}99$ feet.

Second Solution.



Now sin. B = sin. $(180^\circ - \text{B})$
$\text{B} = 180^\circ - 30^\circ\ 39'\ 50'' = 149^\circ\ 20'\ 10''$
and $\text{C} = 180 - 9^\circ\ 20' - 149^\circ\ 20'\ 10''$
$= 21^\circ\ 19'\ 50''$
and $c = 891{\cdot}33$ feet.

Areas.

To find the area of a triangle, when two of its sides and their included angle are given.

Let a and b be the two sides of the triangle, C their included angle; then

$$\text{area} = \tfrac{1}{2}\, a\, b \text{ sine C}$$

Example.—The two sides of a triangle are 1920 and 1152 links and their included angle 53° 8′, required the area.

$$\begin{aligned}
\text{then area} &= \tfrac{1}{2}\, a\, b \text{ sine C} \\
\therefore \text{twice area} &= a\, b \text{ sine C} \\
\text{log. of twice area} &= \text{log. } a + \text{log. } b + \text{L sine C} - 10 \\
& \ 3{\cdot}2833012 \\
& \ 3{\cdot}0614525 \\
& \ \bar{1}{\cdot}9031084 \\
\text{log. of twice area} &= 6{\cdot}2478621 \\
\therefore \text{twice area} &= 1769547 \text{ sq. links} \\
\text{area} &= 884774 \text{ sq. links} = \overset{\text{A.}}{8}\ \overset{\text{R.}}{3}\ \overset{\text{P.}}{15{\cdot}6}
\end{aligned}$$

To find the area of a triangle when the three sides are given.

Let a, b and c be the sides, and s their half sum, then

$$\text{area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}.$$

To find the area of a triangle, when two of its angles and their included side are given.

Let A and B be the two angles, and c their included side; then

$$\text{area} = \frac{c^2 \text{ sine A sine B}}{2 \text{ sine } (A + B)}$$

CHAPTER VII.

SURVEYING WITH THE THEODOLITE.

BOTH in engineering and other surveys the use of the theodolite is often preferable to chain surveying, and in many cases it is absolutely necessary. The chief cases in which it is generally adopted are the following:—

(1) In the surveys of woods, lakes, &c., by ranging a system of lines round the area to be surveyed, and connecting them by taking the angles at their several points of junction.

(2) In the surveys of winding roads, rivers, &c. (This and the first case are technically known as "traversing.")

(3) In determining the positions and distances of several stations, in an extensive engineering or other survey, by taking angles to them from two or more stations the dis tances of which are known. (triangulation).

(4) In the survey of streets of part, or of the whole, of a town.

(5) In railway surveys, &c.

(6) In ranging out a long line over undulating ground.

(7) In a large survey, by dispensing with certain check lines rendered unnecessary by the angular measurements.

(8) In setting out curves.

(9) In taking angles for elevation or depression.

In surveying with the theodolite, the statements made in the chapter on Chain Surveying on the practical application of triangulation, inspection of the ground and setting out, and the general observations, are equally applicable.

The best position for the base line and the principal stations should be carefully selected, and the triangles formed be well-conditioned. In general no angle less than 30° should be used, unless the nature of the ground renders it absolutely necessary. In important surveys the base lines should be measured twice. The stations at the commence-

ment and end of the base line should be easily seen from the surrounding country, and it is preferable that they should be visible from one another. The base line can be lengthened by angular measurement, as will be shown hereafter.

Notwithstanding that, given the measured length of the base and the two angles, the sides of the triangle can be computed by trigonometry; it is necessary, when all the stations are determined, to measure one at least of the distant sides of the system of triangulation, which is known as the base of verification. When the base line and the base of verification have been measured, and the angles of the main triangulation observed, these data should be reduced to paper before the survey is proceeded with. The triangulation being completed, the filling-in presents no difficulty. The larger triangles are subdivided into smaller triangles, and the survey is proceeded with according to the rules of chain surveying, aided in the larger surveys by the use of the box sextant and prismatic compass.

The Methods of taking Angles.

There are two methods of observing angular measurements:

(1) By taking the bearing of two lines formed by the point of observation and two station points.

(2) By taking the bearing which the line formed by the point of observation and the distant station makes with the magnetic meridian, as indicated by the needle of the compass. The first method is more certain, though the two can be used in conjunction.

Bearings with the magnetic meridian can be taken at various stations to some prominent point, visible generally on all parts of the survey, and this arrangement serves as a check on the first method.

Bearings taken with the magnetic meridian are liable to various disturbing influences on the needle, which sometimes considerably affect it. All metallic attractions should be removed from its close proximity.

To take a Horizontal Angle with the Theodolite.

(1) *The bearing of two lines, one with another.* The theodolite being adjusted (as explained in the chapter on Instruments for Angular Measurement), unclamp the vernier

plate, set the arrow of the vernier to 360° or zero on the lower plate as nearly as possible. Clamp the vernier plate, and adjust the points carefully by the microscope and the tangent screw. The other vernier should be examined to ascertain if it reads 180° exactly, or if not so, the difference must be noted. (For the descriptive letters used here the reader should refer to the drawing of the theodolite (page 61) in the chapter on Angular Instruments.)

Again, unclamp the vertical axis by the screw K, and turn the instrument to the left of the two stations, between which the angle is to be taken, till the centre of the cross webs in the telescope nearly cuts the centre of the bottom of the pole set up at the station; then clamp the screw K, and, by gently turning the tangent screw, the most perfect accuracy may be secured.

Next, unclamp the vernier plate, and turn it round till the cross webs nearly cut the bottom of the pole at the second station; then clamp the vernier plate, and obtain the exact point of intersection by the tangent screw as before. Read off the angle in degrees (and half degrees if any) in the scale of the lower limb, as indicated by the arrow of the vernier (page 68), and with the microscope find on the vernier itself which one of its lines coincides with any line on the scale of the lower limb. Read off on the vernier scale the number of divisions or minutes included between the arrow of the vernier and the division so coinciding with the division on the lower limb. (The vernier is fully described in the chapter on Instruments for Angular Measurement.) Then similarly read the angle on the other vernier. Subtract 180° from it, and take the mean of the result and the angle observed with the first vernier. This will give the true reading.

Example.

Reading with first vernier	. . . . .	$=67° 4'$
,, ,, second ,,	$247° 3' = 247° 3' - 180°$	$=67° 3'$
		$2)134° 7'$
	True angle	$67° 3' 30''$

The reason for observing the mean of the readings of the two verniers, is to counteract the effects of eccentricity in the two circular plates and their axes.

To ensure the greatest accuracy, invert the telescope in the Y's (see figure on page 61), and take the mean angle of the two verniers as before. If there is any discrepancy between this result and the first, viz., 67° 3′ 30″, take the mean of the two.

Repeating Angles.—To form a check on the reading, the observation of the angles is repeated in the following manner, and the mean result taken. The horizontal angle between the two points is determined as above. Assume the points to be A and B, the point A being at the left hand, and the angle to be 67° 3′ 30″. Leaving the vernier clamped at 67° 3′ 30″, unclamp the vertical axis by the screw K (see figure on page 61), and direct the telescope on to A again. Clamp the screw K, and fix the exact intersection with the tangent screw—as before. Unclamp the vernier plate, and obtain the intersection of the cross webs of the telescope with the point B, clamping the vernier plate, and using the tangent screw. The angle will now read 134° 7′. If extreme accuracy is required, repeat again, starting from 134° 7′. The next angle will be 201° 10′30″. Divide the total result by the number of readings $\dfrac{201^\circ\ 10'\ 30''}{3} = 67^\circ\ 3'\ 30''$ the true angle. **The reading with the two verniers, and with the telescope reversed, must be done at each repetition.**

(2) *The bearing with the magnetic meridian.*—The instrument being adjusted, unclamp the vernier plate, and set the arrow of the vernier to 360 or zero on the lower plate. Then clamp the vernier plate as before. Unclamp the screw K (see figure on page 61), and move the telescope till the north point of the needle of the compass points exactly to 360° on the card. Clamp the vertical axis by the screw K, unclamp the vernier plate and direct the telescope on to the first station, the bearing of the line between which and the point of observation it is required to ascertain with the magnetic meridian. Clamp the vernier plate, and read off the angle from the two verniers, verifying with the telescope inverted, and repeating as already explained. The angle can also be read on the compass card, but as the diameter is so small it cannot be obtained with accuracy.

To take a Vertical Angle.

Having set the instrument level, and adjusted it as already explained in the chapter on Angular Instruments,

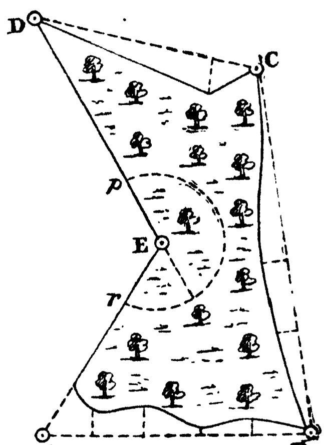
observe whether the zero of the vertical limb coincides with that of its vernier, by the microscope attached thereto. These points being found coincident, raise or depress the telescope, till its optical axis, or cross wires, cut the object required ; then clamp, and adjust till perfect accuracy be obtained, when the angle may be read off, which will be an angle of depression, if the arrow of the vernier be between the zero of the vertical circle and the object glass of the telescope ; otherwise an angle of elevation.

Ascertaining Heights and Inaccessible Points by Angular Measurement.

The reader is referred to the preceding chapter on Logarithms and Trigonometry as applied in Surveying, where this subject is dealt with, and the trigonometrical formulæ given.

To Survey Woods, Lakes, &c., by the Theodolite.

1. Required the plan of the wood represented in the following figure, the field notes being given :—



Fix poles round the wood, so that the lines surrounding it may be as near it as possible, for the convenience of taking the offsets ; the stations at the same time being made on proper ground for fixing the theodolite. Let A B C D E be

the stations, and the field book as below. The lines D E and E A almost impinge on the wood. The long offsets are set out with the optical square.

	to ⊙ C	
10	2678	
101	1400	
119	800	
10	000	
From ⊙ A	81° 29′	to ⊙ C
	⊙ B ⏋	
	to ⊙ B	
10	2302	
99	1800	
10	1100	
202	600	
218	320	
perpr. to corner	225	
Begin at	⊙ A	
		go E

From ⊙ E	58° 23′	to ⊙ B
	to ⊙ A	
	1793	
10	1350	
10	000	
From ⊙ D	241° 38′	to ⊙ A
	⊙ E ⌜	
	to ⊙ E	
10	1790	
10	000	
From ⊙ C	46° 51′	to ⊙ E
	⊙ D ⏋	
	to ⊙ D	
10	1898	
237	200	
10	000	
From ⊙ B	111° 39′	to ⊙ D
	⊙ C ⏋	

It will be seen, from the field notes, that the line A B is first measured, as a base for the plan. The first angle A B C is then taken, and being found to be 81° 29′, shows the direction of the second line B C. The angles at stations C and D are respectively found to be 111° 39′ and 46° 51′, the three lines B C, C D, D E each bending respectively to the left of the line preceding it. The angle at ⊙ E is found to be 241° 38′, which being greater than 180°, that is, greater than the semicircle *p q*, shows that the line E A turns to the right. Finally, the angle at ⊙ A is found to be 58° 23′, showing that the line A B turns to the left of E A. The magnitude of the angle shows whether the new line inclines to the right or the left of the old one, *the new line turning to the left of the old one, when the angle is less than* 180°, *and to the right when greater, the zero of the instrument being always directed to the commencement of the old line.*

Planning and proving the work.—Draw the base line A B in the given direction, and lay off the given length 2302 links thereon. Place the centre of the protractor at ⊙ B, with its straight side close against A B, and prick

off 81° 29′ from its end towards A; then, through ⊙ B and the protractor-mark draw B C, making it the given length 2678. Lay down similarly the two following sides with their angles C and D. Lay off angle at ⊙ E, which is 241° 38′, therefore E A must turn to the right, and E A being drawn, must reach to ⊙ A, where the survey began. If it does not reach to ⊙ A, there has been an error either in taking the angles or in measuring the lines. But since *the sum of all the interior angles of a polygon is equal to twice as many right angles as the figure has sides, lessened by four right angles* (Euc. I. 32. Cor. 1.), and since the given figure has five sides, the sum of all its five interior angles will be $= 5 \times 2 - 4 = 6$ right angles $= 90° \times 6 = 540°$.

This will be found to result by adding all the angles of the figure, as below:—

Angle at B =	81°	29′
—— C =	111	39
—— D =	46	51
—— E =	241	38
—— A =	58	23
Proof, as respects the angles	540°	0′

The above result shows that the angles have been accurately taken; if, therefore, the work does not close, that is, if E A does not reach to ⊙ A, the error is in the measuring of some of the lines, or in making a wrong entry in the field notes.

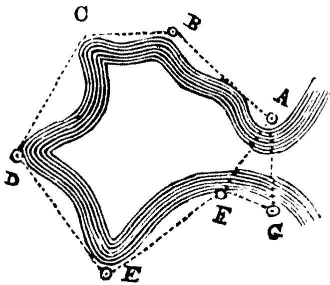
Had the survey been a triangulation of some area instead of a traverse survey as in this case, the sides of the triangle would have been computed by trigonometry and laid down on paper from measures of their length rather than, as already explained, from angular measurements by the protractor, thus ensuring a more accurate result. A check on the accuracy of the angles can also be obtained in a triangulation by observing all the three angles in any triangle; which, if correct, will amount to 180°.

It will be seen that the above survey might have been effected by a chain traverse, as described in Ch. IV., page 50.

2. Lakes, meres, and large ponds are surveyed and planned in the same manner as the wood given above. For instance, the following figure represents a gulf or inlet of the

sea, a plan of which is required to adapt it for the purpose of a harbour for ships.

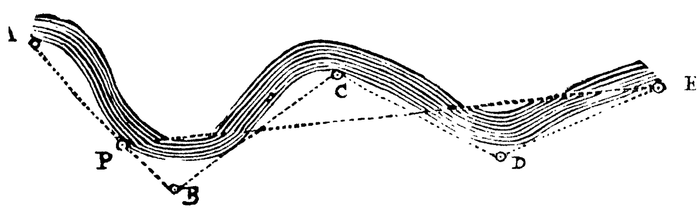
The coast here shown is the boundary of high water. The survey begins at ⊙ A, station points being fixed at B C D E F and G, and angles taken at A to F and G. The line B A, being first prolonged backwards to high - water mark, is then measured to B, and the angle A B C taken. Similarly all the succeeding lines are measured, and the angles taken ; also at F and G the angles are taken to ⊙ A, and the line F G prolonged to high-water mark, all the offsets being taken as the work proceeds.



The figure may now be laid down, precisely as in the last example, the magnitude of the angles showing the directions of the lines, and the lines A F, A G, which could not be measured on account of the great width of the entrance of the harbour, proving the work by means of the angles taken at A and F and G.

The Survey of Roads and Rivers.

1. The following figure represents a river, the plan of which is required for the purpose of improving its navigation, &c.



Stations being set up, at or near the principal windings of the river, as at A B C D and E, the line A B is measured, and the offsets taken to the nearest bank of the river, its opposite bank being determined by a series of lines and angles measured on the opposite side similar to those here

described and connected with them, thus the line B C may be produced for this purpose across the river. A flag is left at ⊙ P in A B, where the sight, in the general direction of the river, is unobstructed for a considerable distance. The measuring of A B being now finished, the angle at B is taken, which, being less than 180°, shows that B C turns to the left. On measuring to C the angle there is found to be greater than 180°, showing that C D turns to the right; and thus the work proceeds to ⊙ E, where an angle is now taken to the now-distant flag at P. This last angle will prove the accuracy of the work when laid down.

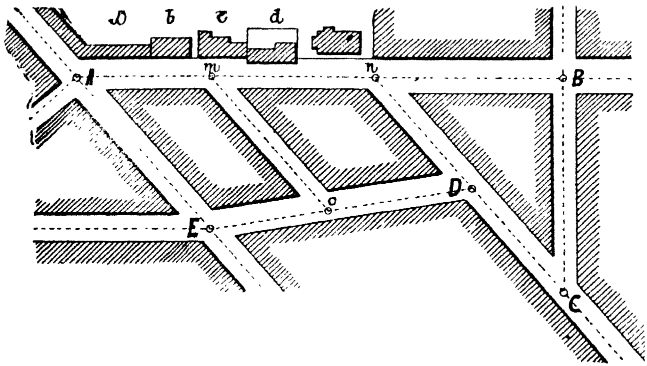
2. If a road be represented by the winding figure in the last example, it may be surveyed in the same manner, excepting that the system of lines A B C D E must be upon the road, instead of the side of it, that the offsets may be readily taken to the right and left of the several lines, recollecting to leave a flag in or near the first line, as at ⊙ P, in the last figure, to which an angle may be taken, after the survey has proceeded a distance, to prove the work.

The Survey of a Part or the Whole of a Town.

Case I.—Commence the survey at the meeting of three or more of the principal streets, through which the longest lines of sight can be obtained, for the purpose of laying out the main lines. Having selected a proper station, fix the theodolite thereon, making a line in one of the principal streets a base line, and directing it to some prominent or well-defined point upon which the theodolite can subsequently be planted. If no natural station point is available, one must be made, and the centre indicated by an iron nail driven into the pavement. All station points must be very carefully noted in the field book, and dimensions taken to each from three or four corners of buildings, or other points, so that the centre of the point of observation can be easily found at any time. Angles must first be taken between the base line and the other lines diverging from the first station, defining carefully the directions of these lines.

This done, measure these lines with the chain, taking offsets to all corners of streets, bends, and to all remarkable objects, as churches, public buildings, &c.; also, defining the extent of buildings belonging to each separate owner,

or joint-owners, if such buildings are required to be taken down for engineering purposes, or for improvements: at the same time recollecting to leave stations opposite the ends of the streets to the right and left, and to take the



angles of the directions. This operation must be repeated on the other main lines, till the survey is completed. It is not sufficient to take offsets only at right angles to the chain. Two offsets at least must be taken to each point from convenient readings on the chain, thus forming a small triangle for each. The points on the chain must be noted at which the sides of buildings at the corners of streets would cut the chain-line if produced. The angles must be taken of the bearings of two lines one with another, and not of the bearing of one line with the magnetic meridian, as this method is unsuitable in town surveying owing to the influence of metallic attractions such as pipes, iron posts, &c., on the needle.

Thus, having fixed the theodolite at A, take the angles of lines meeting there, referring them to the base line A B, that the magnitude of the angle may show their direction: then measure A B, taking offsets to the buildings of different proprietors, as to the buildings marked *a b c*, &c., on which the dimensions of their several parts, yards, &c., must be measured and put in the note book, that they may be accurately mapped, preparatory to their valuation for parish rating, or if required to be taken down for engineering

purposes or for improvements; stations being left at *m* and *n* for the lines in the streets on the right, strong iron pins being driven into the crevices of the pavement for this purpose. The measurement thus proceeds to ⊙ B, where the angles of the streets diverging from it are now taken. The line B C is next measured in like manner, and the angles taken at ⊙ C; after which the measurement proceeds to ⊙ *n* in the base A B, thus constituting the triangle *n* B C; a station being left at D in C *n*. From ⊙ D the survey proceeds to ⊙ E, the angles being taken at E, and from thence to ⊙ A, where the work commenced; a station left in the line D E at *o* for the line to ⊙ *m*. In this manner the survey may be continued to any required extent. It is not necessary to take angles at all stations, as the lines from these are merely filling-in lines between the main lines of the survey.

This survey, so far as it has been here shown, may be plotted independently of the angles taken with the theodolite, by first laying down the triangle *n* B C, and then determining the position of ⊙ E by intersection from stations A and D, when the line *m o* will prove the work. But it rarely happens, in the practice of town surveying, that a fundamental triangle can be obtained sufficiently large to lay down the work in this manner; it is merely here shown that such a case is possible; for had the street in which the line C *n* is measured, been so bent as not to admit of a straight line along it, the use of the theodolite would have been indispensable in this survey. Assuming, therefore, that C *n* is not a straight line, but bent at ⊙ D; then, in the five-sided figure A B C D E, the accuracy of the measurement of the angles may be proved, by taking the sum of all the interior angles, which should equal 540°, viz.: twice as many right angles as the figure has sides less four right angles, and the work further proved by the closing of the lines at ⊙ A as well as at the several other stations.

As a further check on the accuracy of the observation of the angles, the supplement of the angles should be observed also; thus at the station B, assuming the interior angle A B C to be 87° 30′, the exterior angle should be 360° — 87° 30′ = 272° 30′.

The notes in the field book in this case are entered in the same form as those given in the chapter on chain

surveying, excepting that the entries must be made at a sufficient distance apart to allow sketches of buildings, yards, gardens, &c., to be clearly made, with the measures of their several parts put on them in feet and inches.

The chain of 100 feet must be used in all town surveys.

Case II.—If a very large town be required to be surveyed, the best method is to measure a base line of considerable length, on elevated and open ground outside the town, and at two stations at its extremities take horizontal angles to the towers of churches and other lofty buildings in the town, and the intersections of the lines of sight from these angles will determine the positions of the towers, &c.: these may then be made stations for the survey of the several streets, which may now be conducted in the manner shown in Case I. Moreover a third station must be taken in the base line thus measured, at which angles must be taken to all the towers, &c., which angles being laid down, their lines of sight will pass through the intersections of the lines of sight taken from the other two stations, if the work be correct; otherwise an error has been made in taking some of the angles, which must be corrected before the survey of the streets, &c., be commenced.

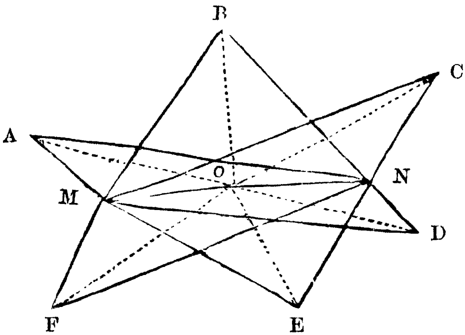
The several distances of the towers and other lofty buildings may be calculated by trigonometry, and the several lines, or triangulation, connecting the said towers, &c., may thence be laid down as shown below.

Case III.—If the town be long and narrow, with straight openings across, either through straight streets or partly through streets and gardens, a triangulation may be formed on the open ground outside the town, and the main lines may be connected by other lines passing through these openings, in which stations may be obtained for the survey of the other streets.

The observation of angles is best effected in the early morning, before the commencement of the traffic, and this remark applies also to linear measurements in the principal streets.

To Determine the Positions of Several Distant Points, by taking Angles at two Stations at the Ends of a Given Line.

Let A, B, C, D, E, F be six stations, the positions of which are to be found. Measure a line M N on level ground, and such that all the stations may be seen from M and N, and at each of the stations M and N, take angles with the theodolite to all the stations; its zero, in taking the angles at each station, being directed to the opposite end of the given line, that the magnitude of the angle may determine the direction of each line of sight to the distant stations.



The line M N being then laid down, and the angles taken at its extremities, the intersections of the lines of sight M A, M B, &c., and of N A, N B, &c., will determine the positions of the several distant points A, B, &c., from whence their distances may be found and other base lines laid down for the further extension of the survey if required.

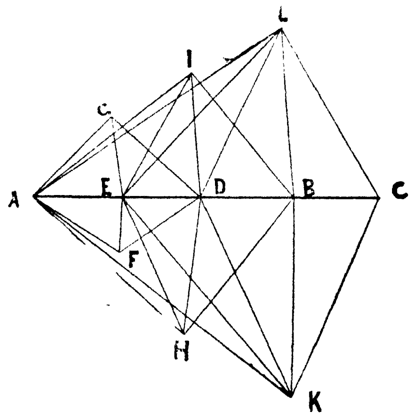
The accuracy of the work may be proved by taking a ⊙ *o*, at any point in M N, and from thence taking angles to all the distant stations; which angles being laid down, their lines of sight will pass through the intersections A, B, &c., if the work has been correctly done.

When all the distant stations or objects cannot be seen from two given stations, then three stations may be taken, or as many as are necessary; connecting the stations thus used, by measuring their distances, and proving their positions by other lines, or by angles: the zero of the theodolite in taking the angles to the distant stations being always directed to the last given station. Moreover, the angles to all remarkable objects that can be seen from two given stations may be taken at the same time; thus may their positions also be determined by the intersection of two

or more lines of sight. In this manner very extensive surveys may be effected, the accuracy of the work being occasionally checked by measuring the distances of some of the distant stations where it can be most conveniently done. The distances from A to B, from B to C, &c., may be calculated by trigonometry.

To Verify and Prolong a Base by Triangulation.

Besides the marks at the extremities of a base line—which, if the base is to form a groundwork for a survey of considerable extent, should be constructed so as to be permanent as well as accurate—intermediate points should be carefully determined and marked during the progress of the measurement, by driving strong pickets or making some other clearly-defined mark, these marks serving for testing the accuracy of the different portions and comparing them with each other.* Thus:—



Let A B represent the portion of the base actually measured, and B C that to be added by calculation for the purpose of extending the base to C, in order to obtain a more eligible termination.

The points E and D having been marked during the measurement, the stations F and G are selected so that the angles at E may be nearly right angles, and the distances E F and E G each about = A E. Similar conditions determine the positions of H, I, K, and L. At A, as well as at every point previously marked on the base, and selected on each side of it, angles are observed to every other point.

With these data, the lengths G E and E F are determined; from each of these E D is obtained by calculation; and from A E and E D and A D as bases, I D and D H are obtained;

* Frome's "Trigonometrical Surveying."

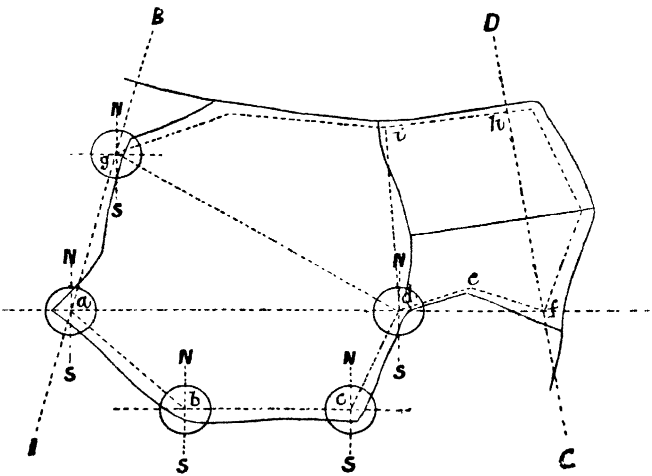
and lastly by similar processes, B L and B K are found as the mean results of many operations all tending to check each other. B C is finally obtained from B L and B K independently used as bases in the triangles B L C and B K C.

In this manner the Irish base on the plain of Magilligan was prolonged about two miles, the termination of the north end of the base being ill-adapted to serve as a station for general observations of the angles.

Surveying with the Magnetic Needle.

The following is an example of a survey made from bearings with the magnetic meridian.*

At *a* let the bearing of *a d* be 91° 5′ east, and the bearing of *a b* 136° east; note this at the commencement of the



line *a b* before you commence chaining. On reaching the point *b* take the bearing of *b a*, and let this be 136° 5′: note this at the end of the line and take 136° 2½′ for the mean, or corrected bearing, east and west of the line *a b*. In the same manner, let the mean bearing of *b c* equal 89° and that of *c d* equal 30°. At *d* take the bearing of *d a* and let this be 91° 10′; then the mean or corrected bearing of *a d* or *d f* will be equal to 91° 7½′. The angle *d a b* is equal to 136° 2½′ minus 91° 7½′, or to 44° 55′. The angle *a b c*

* "Mathematical Instruments," by J. F Heather.

is made up of the angles *a b* N and N *b c*; *a b* N is equal to 180° minus 136° 2½′, or to 43° 57½′; and N *b c* being 89° the angle *a b c* is equal to 43° 57½′ plus 89°, or to 132° 57½′. The angle *b c d* is made up of the angles *b c* N and N *c d*; *b c* N is equal to 180° minus 89° or to 91°, and N *c d* being 30°, *b c d* is equal to 91° plus 30° or 121°. Lastly, *c d a* is equal to S *d a* minus S *d c*; S *d a* is equal to 91° 7½′; and S *d c* being 30°, *c d a* is equal to 91° 7½′ minus 30°, or to 61° 7½′.

Now 44° 55′ + 132° 57½′ + 121° + 61° 7½′ = 360°.

In the same manner we may check the triangle *d e f*, the three angles of a triangle being equal to two right angles. Observe that three more filling-in lines have been run from *g* on A B, to *h* on C D, and that a tie line has been chained from *d* to *i* forming a further check upon the work; for, if in plotting, *d* or *i* lean one way or the other out of position, then the line *d i* will not plot truly.

Latitude and Longitude.—The transit theodolite, being fitted with a vertical circle, and mounted in the same way as a transit instrument, may be used for ascertaining the latitude of any place. (The reader, when perusing this paragraph, may conveniently refer to a Chart of the World, where he will find latitude and longitude marked.)

The meridian of any place is an imaginary circle passing through both poles and the particular place—its plane thus dividing the globe into two hemispheres. It derives its name from the fact that, every place in the globe having its meridian, it is midday or noon when the sun arrives above this circle. It is necessary to keep in mind that a line, called a vertical line, and being perpendicular to the earth's surface at any point, corresponds with the direction of gravity: in other words, if produced inwards it would reach the centre of the earth, and if produced outwards, would cut the heavens in the zenith.

Latitude is the angle which a vertical line at any place makes with the plane of the equator, in the meridian of that place. It is called north, or south, according as the place is situated on either side of the equator. The highest or greatest latitude is 90°—that is, at the poles.

The direction of the true meridian through any point can be ascertained with a transit theodolite as follows:—

When one of the fixed stars is approaching the meridian it will appear to rise until it reaches the meridian;

afterwards it declines. The direction of the true meridian through the place of observation is exactly halfway between any two equal altitudes of the star, before and after culmination. Thus, say a star has an altitude of 50° on the vertical circle when its horizontal angle from a reference point is 62°; also that the star again has an altitude of 50° with a horizontal angle of 102° from the reference point, then the direction of the meridian will be 82° from the reference point. When determining the meridian in this way it is usual to read the horizontal angles for several pairs of equal altitudes, the true direction of the meridian being taken as the mean of that given by the several observations.

Having determined the direction of the meridian the latitude of a place can easily be determined in the northern hemisphere by observation of the pole star when it is on the meridian.

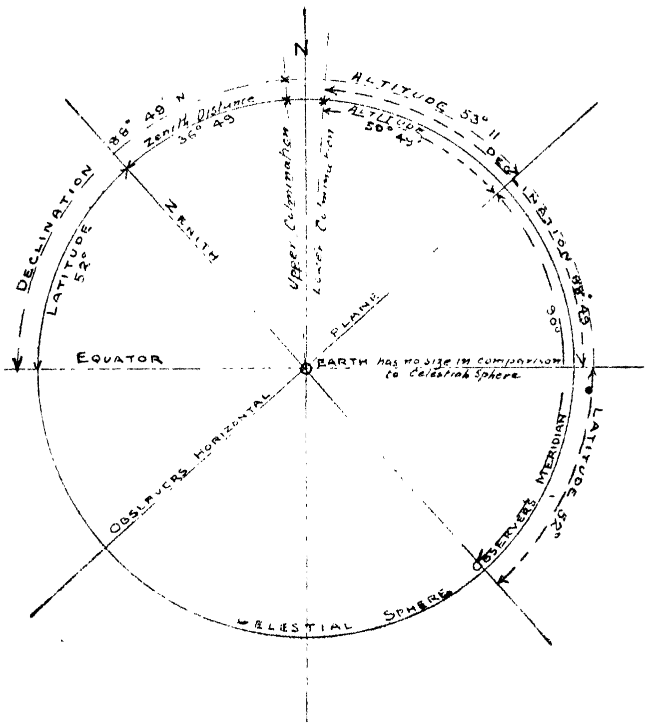
Thus say the observed meridian altitude of the pole star is 53° 11′ 40″ (observed)

Refraction 40″ (Chamber's Tables),

Declination for date 88° 49′ (" Whittaker's Almanac "), then, the true altitude will be 53° 11′ since refraction makes an object appear higher than it really is. Subtracting the true altitude from 90° gives the zenith distance; in our case 36° 49′. Subtracting this from the star's declination gives us 52°, the latitude required. The foregoing assumes that the star is observed at its upper culmination. When it is on the meridian below the pole, add 90° to the corrected altitude and subtract the declination.

Thus, true altitude	50°	49′	
	90	0	
	140°	49′	
	88	49	declination
	52°	0′	latitude.

Knowing the direction of the meridian, longitude can be determined with the aid of a chronometer or a good watch, set to Greenwich time. The time of the sun's passage of our meridian is noted, and the interval of time elapsing between its passage of the meridian at Greenwich (given in the published tables) fixes the longitude. Two successive passages of the apparent sun at Greenwich on he give date is the interval of time corresponding to 360° of longitude. One hour of mean time equals 15° of longitude.



— LATITUDE —

BY MERIDIAN ALTITUDE OF POLE STAR

[*To face page* 98.

CHAPTER VIII.

LEVELLING.

Levelling is defined in the "Imperial Dictionary" as follows: "The art or operation of ascertaining the different elevation of objects on the surface of the earth; the art or practice of finding how much any assigned point is higher or lower than another assigned point." This difference in elevation is determined with reference to an assumed line, called a level line, the extremities of which are equidistant from the earth's centre, and which is called a line of true level. Owing to the figure of the earth being an oblate spheroid, such line is in reality a curve, parallel with the surface of the earth. A line which coincides with or is parallel to the horizon is called a line of "apparent" level.

There are many instruments used in levelling. They can, however, all be reduced to one of five classes, and all classes depend upon one principle, viz., the action of terrestrial gravity. These classes are:—

(1) Those in which the plumb line indicates the vertical line, the horizontal line being at right angles to this. An example of such instrument is the mason's plumb rule.

(2) Those which indicate a horizontal line by the surface of a fluid at rest, free from local disturbing influences. Such conditions are best obtained in a closed tube partially filled with liquid, such as the bubble tube of the spirit level, the bubble of air occupying the highest point of such tube. A simpler example is the water level.

(3) Those in which the difference of level is indicated by measurements of elevation or depression taken by angular instruments, such as the theodolite.

(4) Those in which the difference in level is indicated by the difference in pressure of the atmosphere due to its

density at corresponding altitudes. An example of such an instrument is the barometer.

(5) Those in which the difference in level is indicated by the point of ebullition (or boiling point) of water, which takes place when the tension of its vapour just overcomes the surrounding pressure of the atmosphere. An example of such an instrument is the hypsometer.

Levelling Instruments.

Class I. The Plumb Rule.—The foregoing reference to this is sufficient to show its practical application to this subject.

Class II. (*a*) **The Water Level.**—This is a simple contrivance and can be easily made by anyone. It consists of two vertical tubes connected by a horizontal one, the whole being filled with water, coloured for convenience. The instrument is set so as to turn on a light stand. When the water is at the same height in the two vertical tubes the surfaces are level, and a horizontal line can be sighted with reference to them.

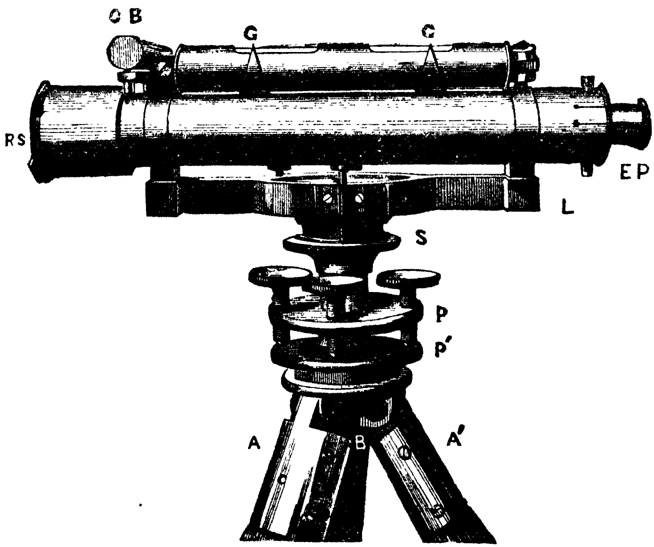
(*b*) **The Mechanic's Level** is the simplest form of spirit level. It consists of a glass tube, filled partially with alcohol, and hermetically sealed. It is impossible to manufacture a tube perfectly straight, and all tubes are placed with the most concave surface internal. The tube is fixed in a hardwood frame with a brass cover. The cover has an opening wherein the bubble can be viewed.

(*c*) **The Surveyor's Level.**—The level more commonly in use is the "dumpy" level, and is a development of that originally designed by Gravatt. A level known as the Y level from its being mounted in Y's, in the same way as the theodolite, is also sometimes more rarely met with in this country. It is the oldest kind of surveyor's level, and has fallen into disuse, owing to the number of its loose parts. It is not proposed in this work to do more than briefly refer to the Y level.

Mr. W. F. Stanley, of Great Turnstile, London, has kindly permitted the drawings of the dumpy level that follow to be reproduced here, and the description thereof is mainly compiled from his work on "Surveying Instruments."

The surveyor's level consists essentially of a telescope with diaphragm at mutual foci of objective and eye-piece, the axis of the telescope being placed in a truly parallel direction with the crown of a sensitive level tube. The whole instrument is adjustable to a position of verticality of its central axis, and horizontality of telescope, in relation to the surface of the earth, in what is termed the setting up of the instrument; so that when it is set up in this position levels may be taken from it in any horizontal direction from one point of observation by rotation of the telescope about its vertical axis.

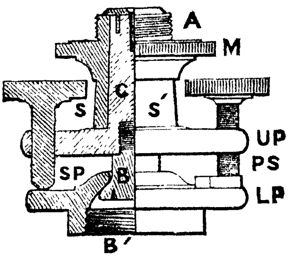
The Dumpy Level consists of a telescope similar to that of the theodolite, fully described in the chapter on angular instruments (see page 64), and carries a ray shade R S at the object-glass end. The eye-piece E P is adjustable to webs



in the telescope by pressure in or out. Two straps or bands are accurately soldered round the tube of the instrument, one of which carries a *hinge-joint* and the other a pair of *locking-nuts* to support the level tube G C, which at the same time permit its adjustment. The lower part of each strap-piece

is left a solid block of metal to give very firm support to the telescope as it rests upon the limb L beneath. The limb may be either a casting with a socket screw only in its centre, or a compass box may be formed in the centre, and the socket screw placed under this as is shown in the figure.

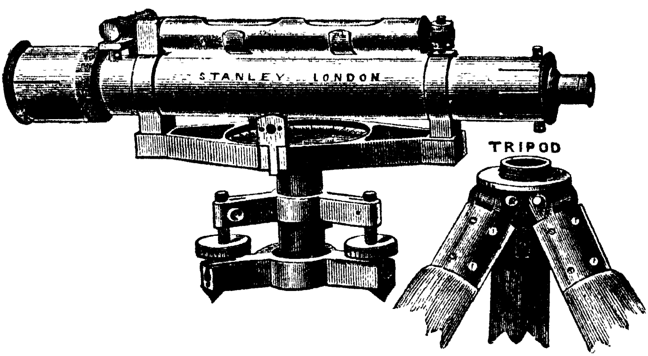
Vertical axis and parallel plates.—A is a screw by which the parallel plate foot is attached to the limb of the instrument; M a large milled head by means of which the screw can be brought up firmly to its collar; S S′ the socket which is ground to fit the cone C; C forms a part of the upper parallel plate U P; B is a ball-pin which screws firmly into C; L P lower parallel plate, part of which forms the lower ball socket, so that the whole instrument rocks about the ball B as a centre by the action of the parallel plate screws P S; B′ female screw for fixing to tripod head. The parallel plate screws are tapped, that is, have female threads cut into the upper plate U P and their points press the lower parallel plate L P at certain points, there being a stop-piece placed round the part of one screw to prevent rotation. The tripod stand is similar to that of the theodolite. The diaphragm of the dumpy level is generally webbed with two vertical webs and one horizontal, but is in other respects similar to that of the theodolite (see page 65), to which the reader is referred. In use the image of the staff, to be described later, is brought between the vertical webs which indicate whether it is held upright. The upper margin of the portion of the horizontal web between the two vertical ones is the index of level to which all readings are made either for the adjustment or for reading the levelling staff in the field. The diaphragm of the level is adjustable by two vertical screws.



Subtense or stadia webs.—It is very advisable in all levels to have two extra webs (or lines, cut in the glass diaphragm) placed one on each side of the central horizontal web, or line, fixed at such distance apart that the image of 1 foot of staff enclosed between the lines represents a distance of 100 feet from the nodal point of the instrument. The

latter is a point in front of the object glass, and its distance from the centre of the instrument is supplied by the maker. This is known as the constant, and must always be added to the distance read from the staff. Thus, say the lower stadia line reads 1·33 feet on the staff, the middle web 3·62, and the upper stadia line 5·91; the centre reading is the height of the line of collimation, 5·91 — 1·33 = 4·58; say the instrument constant is 12 inches. Then the distance from the centre of the instrument to the staff is 458 + 1 = 459 feet

Improved Dumpy Level.—An improved dumpy level is now made of which the vertical axis is fixed directly upon the limb, and not through a loose screw fitting for separation at this point as in the ordinary dumpy. The setting-up adjustment is upon tribrach limbs with three screws only. These screws never strain the vertical axis. A level tube more sensitive can be put in than in the ordinary dumpy. Instead of the horizontal line forming the web of the diaphragm, a finely-pointed platino-iridium needle existing half-way across the diaphragm indicates the horizontal line of collimation by means of a point.



The adjustment of the level over the three screws is here inserted for convenient reference.

Setting up level in tribrach system of improved dumpy.—The level is first set parallel with any two of the screws, and the long-bubble adjusted by these screws to the centre of its run. The telescope is then revolved until it is directly over the remaining screw. The bubble is again brought to centre with this screw only. These operations are repeated

once or twice until the bubble remains centred with the telescope pointing in any direction.

A three-screw instrument cannot be levelled by placing the telescope over each of the three screws in turn. Placing the telescope first parallel to two screws and then turning it through a right angle over the third screw is imperative. A small cross-bubble is fitted to facilitate setting the instrument approximately level with the stand.

The adjustments of the ordinary dumpy level are—

(1) Adjustment for parallax.
(2) Levelling the plates by the parallel screws.
(3) Adjustment of the horizontal limb.
(4) Adjustment of the line of collimation.
(5) Adjustment of the vertical limb.

The adjustments 1 and 2 must be made on each occasion the level is used. The adjustments 3, 4 and 5 are made by the instrument maker before the level leaves his works. The engineer may attend to Nos. 3 and 4, but in case the adjustment No. 5 is required, the instrument should be sent to the maker.

(1) *The adjustment for parallax* is the same as that described for the theodolite in chapter on angular instruments (p. 65).

(2) *The levelling of the plates by the parallel screws* is also similar to that described for the theodolite with the exception that the level tube G C in the illustration, page 101, is used instead of the levels O and I (Fig., page 61).

(3) *The adjustment of the horizontal limb.*—Place the telescope over two of the parallel screws and bring the bubble of the level by means of them to the centre of the run. Turn the telescope 180°, so that the eye-piece rests in the same position as the object glass did on the first placing of the telescope. If the bubble is not now in the centre of its run correct for half the error by the parallel screws over which it rests, and for the remaining half error by the hinge-joint and locking-nuts which support the level on the telescope. Repeat the operations till the bubble remains in the centre of its run in both positions of the telescope. Then turn the telescope over the other pair of parallel screws, and perform the same adjustments as described, The bubble should now retain its position in any direction the telescope is turned, showing that the internal axis about which it turns is truly vertical. If it does not, and

if the error is serious, the instrument should be sent to the maker, and the adjustment No. 5 referred to, be made by him as already explained.

(4) *Adjustment of the line of collimation.*—The line of collimation has been fully dealt with in the part of this work referring to the theodolite. The mode of adjustment in the two instruments varies, that for the dumpy level being as follows :—On a tolerably level piece of ground measure 100 feet exactly, and mark with a peg A and C on each side of a central peg B thus :—

A		B		C
⊙		⊙		⊙
	100 feet.		100 feet.	

The pegs are to be driven down, so that the upper surfaces are level, which must be ascertained with an ordinary mechanic's level. The adjustment of the horizontal limb as described above must be examined, and if not accurate made so. Then plumb the level exactly over the centre peg, and take a reading from a Sopwith's levelling staff (as described later) on the peg A. Place the same staff on C and take the reading of this. Raise or lower, as may be required, the peg A or the peg C, until the reading from point B to each is the same. This gives a perfect level. Whatever error there is in the diaphragm, being equal for each distance A to B and B to C will not affect the case. Next take the level and adjust it at a point D in line with the other pegs, and 20 feet further from B than the point A, as shown thus :—

D		A		B		C
⊙		⊙		⊙		⊙
	20 feet.		100 feet.		100 feet.	

Then with the level at D take the readings on the pegs A and C. These should be exactly alike, and if not the line of collimation is in error. **The collimating of a level of this type is a somewhat complicated matter, so it has been thought advisable to give a full description in the form of an appendix. (See p. 235, Appendix V.)**

Levelling with the Mechanic's Level.—This is the simplest form of levelling, and is commonly done in build-

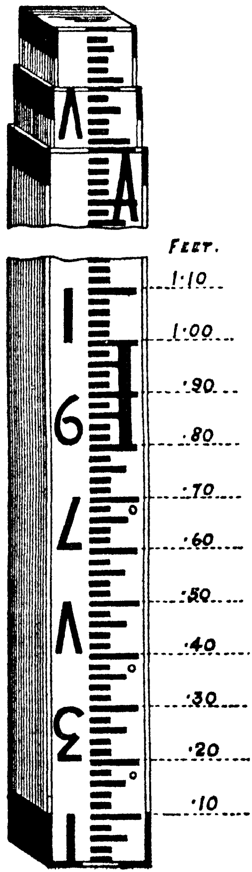
ing operations. A wooden straight edge 9 to 10 feet long about 4 inches wide and $\frac{3}{4}$-inch thick, to prevent warping, is used. It should have its faces and edges planed straight and true. The level is laid on the centre of the straight edge, one end of which is set on the place, with reference to which the level of a further point is required. The straight edge is raised at one end until the bubble is in the centre of its run. The level is then reversed to see if it is true. If there is a difference between the positions of the bubble tube of the level in its two positions, the straight edge should be finally fixed so that the mean of the two positions of the bubble may be taken. The two points, on which the straight edge rests, are now level, and any number of points may be found by continuing the process described.

Boning Rods.—A quicker method of the simple class of levelling is effected by the use of boning rods These are three in number exactly similar, usually 3 feet long, 3 inches wide, and $\frac{1}{2}$-inch thick. They have a cross head about 6 inches long. Paviors use them, and for draining they are indispensable, but are in this case usually of greater length than 3 feet. The method of using them is as follows :—

One boning rod is set up at the starting level point, and another at such a distance that the straight edge described above can conveniently rest on it. The second boning rod is raised or lowered as required until the straight edge is level, as indicated by a mechanic's level. A third boning rod is then set up at a similar distance, and the straight edge set upon the second and third boning rods, but so that that part of the level which, when set up between rods 1 and 2, pointed to rod 1, now points to rod 3. The third rod is raised or lowered until the straight edge between 2 and 3 is level. A line now joining the tops of 1 and 3 will be truly level, as any error in the level has been neutralized by the reversing process. between rods 2 and 3. The rod 2 can now be removed, and can be set up at any distance, or boned, as it is termed, so that the top of it is in line with the tops of Nos. 1 and 3, forming a level line. If a point say 6 inches below or above the starting point is desired, the line is boned level as described, and the third rod then raised or lowered the required distance.

Any intermediate points between the first and third rods can be found by boning in between these points, so that the tops of the three boning rods are always in a straight line.

Levelling with the Surveyor's Level. The Levelling Staff.—With the surveyor's level the vertical distances are determined from the surface of the ground to the point on a staff, resting on such surface, at which the cross web of the diaphragm, being in the optical axis, or line of collimation of the level, cuts the staff.



There are many kinds of such staves, but that now generally in use in this country was invented by Sopwith, and is known as the Sopwith staff. It is 14 feet in height, divided into three parts, sliding together, the two upper fixed by spring catches. The staff is divided into hundredths of a foot, each hundredth alternately coloured black and white. The feet are shown in large red figures, and the tenths by large black ones.

The illustration will show the method of reading the staff. In some staves, and with great advantage for reading at close distance, small red figures of the number of feet are placed between the large red ones. Staves can be had either with the measurement as above painted on, or with papers so marked pasted on the staff. The painted staves are recommended as more accurate, and as wearing better and keeping cleaner.

CHAPTER IX.

LEVELLING (*continued*)

Curvature and Refraction.—In levelling there are two important elements to be considered, which if not taken into account will, in extended operations, result in serious error. They are—

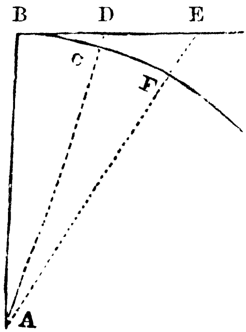
(1) The curvature of the earth.
(2) Terrestrial refraction.

It will be shown later how in ordinary practice the possible errors arising from these causes can be very simply eliminated without calculation, but occasions may occur, as in the following examples, when corrections of apparent level must be undertaken.

The curvature of the earth.—At the beginning of the last Chapter (p. 99) it was shown that, owing to the figure of the earth being that of an oblate spheroid, a line of true level is in reality a curve, the extremities of which are equidistant from the earth's centre. The difference between the apparent level, as indicated by the line of sight of the telescope of the level, and the line of true level, or line parallel with the contour of the earth, is $\frac{1}{100}$ of a foot at a distance of 650 feet from the instrument. The following demonstration will more fully explain the source of error, and the correction to be applied.

Let B D E be a horizontal line—that is, such as would be given by the line of sight of a level properly adjusted ;

B C F an arc of a great circle of the earth; and A its centre.



It will at once appear from the figure, that the heights D C, E F of the apparent level B E, above the true level, increase successively from the point B. The height E F of the apparent level above the true, is equal to the square of the distance B E divided by twice the earth's radius A B, that is $E\ F = \frac{B\ E^2}{2\ A\ B}$, and similarly $D\ C = \frac{B\ D^2}{2\ A\ B}$, &c., therefore the corrections for curvature, D C, E F, &c., vary as the squares of the distances B D, B E, &c., since 2 A B is a constant quantity.

Correction for curvature.—Taking the earth's radius to be $3956\frac{1}{2}$ miles, and assuming the distance B D to be 1 mile, then the correction for curvature $D\,C = B\,D^2 \div 2\,A\,B = 1^2 \div 7913 = \frac{1}{7913}$ of a mile $= 8{\cdot}007$ inches = nearly 8 inches. If the distance B E = 3 miles, then the correction $E\,F = B\,E^2 \div 2\,A\,B = \frac{9}{7913} = 72{\cdot}0637$ inches, or more than 6 feet.

Let any distance B D = d in miles, and the correction for curvature for 1 mile be taken = 8 inches = $\frac{2}{3}$ of a foot, which it is very nearly; then

$$\text{correction} = \frac{2\,d^2}{3} \text{ feet,}$$

for any distance d in miles;

and let $\frac{c}{80} = d$, c being chains; then

$$\text{correction} = \frac{2\,d^2}{3} = \frac{2 \times 12\,c^2}{3 \times 80^2} = \frac{c^2}{800} \text{ inches.}$$

for any distance c in chains.

Terrestrial refraction.—Refraction is "the term generally applied to change of direction impressed upon rays of light traversing a medium the density of which is not uniform, as for instance the atmosphere. Terrestrial refraction is that refraction which makes terrestrial objects appear to be raised higher than they are in reality. This

arises from the air being denser near the surface of the earth than it is at a higher elevation, its refractive power increasing as its density increases."*

Correction for Refraction. — The correction for refraction varies with the state of the atmosphere, but it may generally be taken at $\frac{1}{7}$ of the correction for curvature, as an average; and since refraction makes objects appear higher than they really are, the correction for it must be deducted from that for curvature.

Example 1. Required the correction for curvature and refraction, when the distance of the object is $2\frac{1}{2}$ miles.

$$\frac{2}{3} \times (2{\cdot}5^2) = \frac{2 \times 6{\cdot}25}{3} = 4{\cdot}166 \text{ cor. for curvature.}$$

$\frac{1}{7}$ of which is .... ·595 cor. for refraction.

Difference....................3·571 feet, cor. required.

Example 2. Required the correction, as in the last example, when the distance is 60 chains.

$60^2 \div 800 = 4{\cdot}5$ cor. for curvature.

$\frac{1}{7}$ of which is ·643 cor. for refraction.

Difference ... 3·857 inches, cor. required.

Example 3. From a point in the Folkestone road, the top of the keep of Dover Castle was observed to coincide with the horizontal wire of a levelling telescope, when adjusted for observation, and therefore was apparently on the same level; the distance of the instrument from the castle was $4\frac{1}{2}$ miles. Required the correction for curvature and refraction, that is, the true height of the keep of the castle above the point of observation.

$$\frac{2}{3} \times (4{\cdot}5)^2 = \frac{40{\cdot}5}{3} = 13{\cdot}5 \text{ feet, cor. for curvature.}$$

$\frac{1}{7}$ of which .... = 1·93 feet, cor. for refraction.

Difference .... = 11·57 feet, cor. required.

The Table set out on the following page shows the allowance to be made for curvature and refraction for distances varying from $\frac{1}{4}$ mile to 30 miles :—

* "Imperial Dictionary."

Table of Allowances for Curvature and Refraction.

Difference between apparent and true level for distances in miles. Correction in feet and decimals.

Dist. in miles.	For curvature.	For refraction.	For curvature and refraction.
¼	·04	·01	·03
½	·17	·02	·15
¾	·37	·05	·32
1	·67	·10	·57
1½	1·50	·21	1·29
2	2·67	·38	2·29
2½	4·17	·60	3·57
3	6·00	·86	5·14
3½	8·17	1·17	7·00
4	10·67	1·52	9·15
4½	13·50	1·93	11·57
5	16·67	2·38	14·29
5½	20·17	2·88	17·29
6	24·00	3·43	20·57
6½	28·17	4·02	24·15
7	32·67	4·67	28·00
7½	37·50	5·36	32·14
8	42·67	6·10	36·57
8½	48·17	6·88	41·29
9	54·00	7·71	46·29
9½	60·17	8·60	51·57
10	66·67	9·52	57·15
10½	73·50	10·50	63·00
11	80·67	11·52	69·15
11½	88·17	12·60	75·57
12	96·00	13·71	82·29
12½	104·17	14·88	89·29
13	112·67	16·10	96·57
13½	121·50	17·36	104·14
14	130·67	18·67	112·00
14½	140·17	20·02	120·15
15	150·00	21·43	128·57
15½	160·17	22·88	137·29
16	170·67	24·38	146·29
16½	181·50	25·93	155·57
17	192·67	27·52	165·15
17½	204·17	29·17	175·00
18	216·00	30·86	185·14
18½	228·17	32·60	195·57
19	240·67	34·38	206·29
19½	253·50	36·21	217·29
20	266·67	38·10	228·57
21	294·00	42·00	252·00
22	322·67	46·10	276·57
23	352·67	50·38	302·29
24	384·00	54·86	329·14
25	416·67	59·52	357·15
26	450·67	64·38	386·29
27	486·00	69·43	416·57
28	522·67	74·67	448·00
29	560·67	80·10	480·57
30	600·00	85·71	514·29

Difference between apparent and true level for distances in chains. Correction in decimals of feet.

Dist. in chains.	For curvature.	For refraction.	For curvature and refraction.
3½	·001	·000	·001
4	·002	·000	·002
4½	·002	·000	·002
5	·003	·000	·003
5½	·003	·000	·003
6	·004	·001	·003
6½	·004	·001	·003
7	·005	·001	·004
7½	·006	·001	·005
8	·007	·001	·006
8½	·008	·001	·007
9	·008	·001	·007
9½	·009	·001	·008
10	·010	·001	·009
10½	·011	·002	·009
11	·013	·002	·011
11½	·014	·002	·012
12	·015	·002	·013
12½	·016	·002	·014
13	·018	·003	·015
13½	·019	·003	·016
14	·020	·003	·017
14½	·022	·003	·019
15	·023	·003	·020
15½	·025	·004	·021
16	·027	·004	·023
16½	·028	·004	·024
17	·030	·004	·026
17½	·032	·005	·027
18	·034	·005	·029
18½	·036	·005	·031
19	·038	·005	·033
19½	·040	·006	·034
20	·042	·006	·036
20½	·044	·006	·038
21	·046	·007	·039
21½	·048	·007	·041
22	·050	·007	·043
22½	·053	·008	·045
23	·055	·008	·047
23½	·058	·008	·050
24	·060	·009	·051
24½	·063	·009	·054
25	·065	·009	·056
25½	·068	·010	·058
26	·070	·010	·060
26½	·073	·010	·063
27	·076	·011	·065
27½	·079	·011	·068
28	·082	·012	·070
28½	·085	·012	·073
29	·088	·013	·075
29½	·091	·013	·078
30	·094	·013	·081

Procedure in Levelling with the Surveyor's Level.—Levelling operations with this instrument are divided into—

(1) *Simple levelling*, which consists in ascertaining the difference in level of one point from another, the point being located, so that both can be seen from the place where the level is set up.

(2) *Compound levelling*, of the same class as the first, but extended, consisting, in fact, of several simple levelling operations connected together.

Ordinary examples of compound levelling are—

(1) The taking of "flying levels," or the ascertaining of the levels of the various points in a line of country as a rough guide to a general configuration of the surface, and also to determine the levels of certain points to serve as starting points for future levelling operations.

(2) The taking of levels over what is known as a section line (defined later), such line being previously set out over country already delineated on a plan, the horizontal distances being measured at the same time as the levelling operations proceed. Ancillary to this is the taking of "cross-sections," or the ascertaining of the level of the surface for a required distance on one or both sides of the section line at certain fixed points.

(3) Contouring. This is defined and separately dealt with in the latter part of this chapter.

Record of Results of Levelling Operations.—Levels can be recorded in several ways:—

(1) In a book for future reference.

(2) On a plan, the point at which a level is taken being marked, and the reduced level written adjoining it.

(3) On a section (to be explained hereafter).

(4) On a plan by means of contour lines.

Curvature and Refraction in Ordinary Operations.—In ordinary operations corrections for curvature and refraction are seldom applied, the level being usually placed midway between the stations the levels of which are to be observed; hence the resulting corrections for each station are equal, and the difference therefore of the levels at the two stations is as truly shown by the difference of the readings of the two staves fixed thereon as if the

corrections had been made. Thus the trouble of making these corrections is avoided by *simply placing the instrument midway between the two staves.* In practice, it happens from time to time that, owing to the nature of the ground, it is impracticable to plant the level midway between the two stations. In such case—remembering that a distance of 650 feet produces an error of ·01 foot owing to curvature —the station should not be far from the level, and the accuracy of the observation will be unimpaired.

Precautions in Levelling.

The greatest care is required in levelling, both on the part of the observer at the instrument and of the man who holds the staff. The observer should explain to the staff-holder the important part he serves by the proper discharge of his duties. He should be shown that each of the telescopic staves must be drawn out to its full length, and told to keep his eye on the spring catch securing them. He must always hold the staff in a vertical position, and in windy weather it requires some attention to do this. He should also be shown how to select suitable station points, such as stones firmly fixed in the road away from the main traffic, if levelling is being conducted along such a road.

In the case of levelling through the open country he should be provided with a small circular footplate with convex boss, which any blacksmith can make. When he has once held the staff on the station point he must not leave it without the express direction of the observer at the instrument. The observer should read each station twice at the least, and the custom of the author is to read it three times. In the case of an intermediate sight it is not necessary to read more than once, excepting in the event of desiring to ascertain the level of a fixed point, such as a doorstep, or some other point which the observer desires to set up for future reference. Such fixed points are technically termed "bench marks," as distinguished from those of the Ordnance Survey, which are known as Ordnance bench marks. When reading stations and such intermediate points the bubble should be examined to see if it is in the centre of its run, and if not, made so, prior to taking the reading. In examining the final position of the

bubble, the observer should stand at right angles to it. All readings should be made with the top edge of the exact centre of the horizontal web, as in some levels, although the centre is the line of collimation, the web is not exactly horizontal.

Levelling should not be done on very windy days, as the same standard of accuracy cannot be secured, and similarly not in the rain, as the lenses are obscured, and sometimes the webs of the diaphragm are broken.

Simple Levelling.

In the description of the dumpy level it was explained that the point at which the optical axis—or line of collimation of the instrument as indicated by the horizontal web of the diaphragm—cuts the image of the staff, was the index of level.

To find the difference of level between two points A and C, set up the level at a point B equidistant from A and C, as nearly as can be judged by the eye. Adjust the level and correct for parallax. The rule for difference of level is "more staff, fall in ground; less staff, rise," since in the first case the ground is farther from the line of collimation, and in the second case less, the line of collimation being level.

Example 1.

Reading on staff at A as found	is	10·66
Reading on staff at C ,,	is	11·78 (or more staff)
Difference in level or fall between A and C		1·12 feet

Example 2.

Reading on staff at A	9·66
Reading on staff at C	4·32 (or less staff)
Difference in level or rise between A and C	5·34 feet.

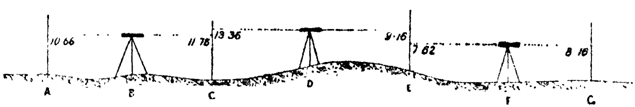
Compound Levelling.

Flying levels.—To find the difference in level between points A and G, set up and adjust the level at B at conveni-

ent distance from A and take the reading on A, which is in this case 10·66. Enter those figures in the column of back sig s in the level book as in that accompanying this example (page 116). Direct the staff holder to select a suitable station at C, at which point take the reading 11·78 which will be a fore sight, and enter it in the book. In all cases the first reading on setting up the level is a "back" sight, and the last reading before removing the level to fresh ground a "fore" sight.

It will be seen that the distances in height at points A and C from the line of collimation, when the level is at B, will give the difference in level between these two points, as in the case of simple levelling; the heights, 10·66 at A, and 11·78 at C, give more staff at C = fall of 1·12 feet.

Remove the level, and adjust it at D, the staff holder still remaining at C, but turning the face of the staff towards the



level at D. With the level at D, take the reading at C 13·36 which will be a back sight as explained. Place these figures in the column of back sights in the level book in a line with the last fore sight 11·78 as shown. Take the reading at a suitable station E 9·16 selected by the staff holder. This will be entered in the column of fore sights, and gives (less staff) a rise of 4·20 above point C or (rise 4·20—fall of 1·12) a total rise at E of 3·08 above A.

Remove the instrument to F, the staff holder remaining at E, but turning the face of the staff as before. The back sight at E is read to be 7·62. Direct the staff holder to G and take a fore sight, which is found to be 8·16 or a fall of 0·54 from E, giving a total rise of (3·08 – 0·54) to 2·54 from point A. The results of these levelling operations are set out in detail below.

Back sight on staff A	10·66 feet
Fore sight on staff C	11·78
The fall from A to C	1·12 difference.

Back sight on staff C	13·36	
Fore sight on staff E	9·16	
The rise from C to E	4·20	difference.
Subtract the fall from A to C...	1·12	
The rise from A to E	3·08	difference.
Back sight on staff E	7·62	
Fore sight on staff G	8·16	
The fall from E to G	0·54	difference.

This fall taken from the rise from A to E, that is,

3·08
0·54

gives the total rise from A to G ... ... 2·54, or nearly 2 feet 6½ inches.

The difference of the sums of the back and fore readings of the staves, will more readily give the difference of level.

Back sights.	Fore sights.
feet.	feet.
10·66 at A	11·78 at C
13·36 at C	9·16 at E
7·62 at E	8·16 at G
sums 31·64	29·10
29·10	
2·54 difference of level.	

Copy of Level Book to illustrate example of Flying Levels.

Back Sights.	Intermediates	Fore Sights.	Rise.	Fall.	Reduced Levels.	Reading on Chain.	Remarks.
10·66							Point A
13·36		11·78		1·12			Point C
7·62		9·16	4·20				Point E
		8·16		0·54			Point G
31·64		29·10	4·20	1·66			

The levels in the foregoing level book are not "reduced" —that is to say, reduced to some given datum - and they are not self checking, so that if an error has been made in any of the readings there is no means of detecting it. A complete system of keeping the level book and of checking the levels is explained later, this example being merely an illustration of the principle.

A Section is a representation of the surface of the country or other object, as it would appear if cut through by an intersecting plane showing the internal structure.

Sections are divided into longitudinal- and cross-sections, the longitudinal being a section cutting through an object lengthwise and vertically, the cross- (or transverse) section cutting crosswise and vertically. A section line is the line of the intersecting plane referred to.

Datum.—All levels are estimated with reference to a horizontal line, either real (such as sea level) or imaginary, such as a line at a level of 100 feet above sea level—or, in another assumed case, 20 feet below the threshold of a certain building. Such line, real or imaginary, is known as a datum, the line of collimation of the level, when adjusted, being of course parallel with it.

Ordnance Datum.—The Ordnance datum is the datum to which the levels on the Ordnance surveys are referred.

For Great Britain it is the mean level of the sea at Liverpool, which is 0·65 of a foot below the general mean level of the sea as ascertained from tidal observations taken at 32 stations in England and 18 in Scotland.

The Ordnance datum for Ireland is not the same as that for Great Britain. It is indicated by a mark fixed in 1837 on Poolbeg lighthouse, in Dublin Bay, and it represented at that time the low-water mark of spring tides.

Any ascertained level relative to the Ordnance datum is marked thus ⩚ on some permanent object, such as a bridge, town hall, milestone, or other suitable place. On referring to the Ordnance Map relating to the particular part of the country, the height above the Ordnance datum is indicated thus, for example, ⩚ B. M. 134·60, showing that the centre of the horizontal line of the mark ⩚ at the point in question is 134·60 feet above the Ordnance datum, the letters B. M. standing for Ordnance bench mark.

Bench Mark, in ordinary practice, and distinguished from Ordnance bench mark, is a datum point fixed on the ground by the leveller for convenient reference, if occasion should present itself for any further levelling in the particular locality after the country has been already levelled. This saves the trouble of re-levelling the whole length, and also acts as a check on the levels. As previously stated it may be a threshold or other known point of sufficient permanency. In selecting the place for a bench mark, the highest point should always be taken, and a sketch made in the level book indicating, and describing fully, the particular point thus :—" B. M. on the south corner of top step No. 55, Station Road."

In selecting a step for a bench mark it should be very carefully examined, as steps are often worn and of uneven level.

In the case of a bench mark being desired in open country, the leveller should cut one on a large tree thus $\overline{\uparrow}$, taking the same precaution in describing it as stated above, for example :—" Bench mark thus $\overline{\uparrow}$ on oak tree, north corner of plantation at point A on plan."

In levelling operations bench marks should be fixed at every quarter-mile at least, and oftener when necessary, in case of continued levelling being required, as for instance in setting out sewers.

The Level Book.

In the author's opinion the best form of level book is one 4 inches broad and 7 inches long, opening lengthwise. This saves labour in adding up the respective columns. The form of level book (pp. 121, 122) used in taking levels for the section referred to below will illustrate this description.

It will be seen that there are seven columns and a space for descriptive observations. The first three columns are used for entering the vertical heights, observed at the time of levelling :—the back sights, the intermediate sights, and fore sights. The back and fore sights have been dealt with when describing simple levelling. The column for intermediate sights is for such sights, the levels of which are read between those read when the level is first adjusted, and the final reading before moving the instrument to fresh ground. The last reading on each page should be a fore sight for convenience in checking and "reducing" the levels. The next two columns are for the rise and fall in the sights as already explained. The sixth column is for the "reduced"

levels, or levels reduced to some known datum fixed at the commencement of the section. The seventh column is for the horizontal distances measured.

Levelling along a Section Line.

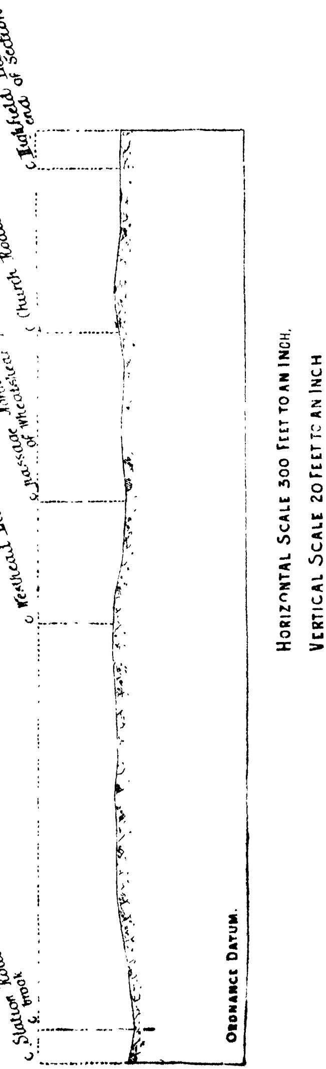
In this case the horizontal distances are measured at the same time that the levelling is done, the line of section having been previously set out. The levels should commence at a bench mark, the level of which is known, and the operations should be continued to another known bench mark. It is not necessary that these bench marks should be Ordnance bench marks (since, in order to attain this, levelling operations might have to be extended for a considerable distance), but it is desirable that they should be.

A proof of the accuracy of the levelling is thus arrived at, since the distance between the sum of the back sights and the sum of the fore sights, if the levels have been accurately taken, should equal the difference in level of the datum points at the commencement and end of the section. Frequently the levelling operations, commencing on a certain bench mark, can conveniently be arranged to end on the same bench mark. In this case no difference should exist between the sum of the back sights and the sum of the fore sights. A reading is taken at each appreciable change of inclination of the ground.

In the case of a ditch and fence it is usual to take a reading of the surface on each side of the fence, a reading in the bottom of the ditch, and a reading of the top of the bank or cop on which the hedge is planted. If, when levelling, the level of an intermediate point (not required for a bench mark) is too low to be read by the level, the staff may be held up till the line of collimation cuts it. The reading is then taken, and the distance from the foot of the staff to the ground at that particular point is added to the staff-reading, thus $13{\cdot}70+5{\cdot}04=18{\cdot}74$.

The copy here given (pp. 121, 122) of the level book used by the author in taking measurements and levels for the main sewerage works of a parish in Lancashire will, it is hoped, together with the section likewise given (p. 120) render the method of procedure intelligible. The horizontal scale of the section is reduced for convenience; the actual scale to which the section was drawn was 1 chain per inch. Another example of a section is given in Chapter X., in which the line is taken across country.

Section taken for Main Sewerage of Croston.



Level Book (Main Sewerage of Croston, June, 1897).

Back Sights.	Intermediate Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Chain.	Remarks.
2·71					24·6	feet.	O. B. M. on cottage abutting on brook and opposite gas-works
	7·73			5·02	19·58		Top of 6-in. common tile in footpath, just before entrance to brook, being drain for houses from Station to point A on plan.
	7·10		0·63		20·21		Top of 6-in earthenware drain in footway before entrance to brook, draining houses from point A to brook.
	8·45			1·35	18·86		Invert of 9-in. E.S.P. drain, 89 ft. 3 in. distant from C. Station Road, draining the Trafford Arms and building land on W. side Station Road.
	3·74		4·71		23·57	00	C. Station Road.
	4·85			1·11	22·46	100	C. roadway over brook, 3 ft. wide.
	6·82			1·97	20·49		Underside stone bearer Culvert under road.
	8·88			2·06	18·43		Bottom of brook under road.
	4·05		4·83		23·26	200	In C. Road Cannel Leach.
6 97		3·21	0·84		24·10	300	Ditto.
	5·92		1·05		25·15	400	Ditto.
	5·32		0 60		25·75	482½	Ditto, in line with N gable, Plough Inn.
	5·06		0·26		26·01		Threshold, ditto
					23·76		Cellar, 2 ft. 3 in. deep below.
	5·25			0·19	25·82	600	
	2·58		2 67		28·49		Top threshold, Crown Hotel.
					23·24		Cellar 5 ft. 3 in. below
4·67		5·21		2·63	25 86	700	
						712·6	In line with S. gable, Crown Hotel.
	4·40		0·27		26·13	800	
	4·70			0·30	25·83	900	
	4·70			0·00	25·83	1000	
5·60		4·51	0·19		26·02	1100	
	5·28		0·32		26·34	1200	
						1269/9	In line with S. gable, Railway Tavern.
	4·94		0·34		26·68		Threshold, Railway Tavern.
					23·38		Cellar 2 ft. 10 in. below
	5·02			0·08	26·60	1315	C. Westhead Lane.
	5·55			0·53	26·07	1400	
		7·04		1·49	24·58	1500	
19·95		19·97	16·71	16·73			

Level Book—*continued.*

Back Sights.	Inter-mediate Sights.	Fore Sights.	Rise.	Fall.	Reduced Level.	Chain.	Remarks.
						feet.	
19·95		19·97	16·71	16·73	24.58		Brought forward.
3·90							
	4·55			0·65	23·93	1600	
						1683	Opp. C. lobby, N side Wheatsheaf.
	4·75			0·20	23·73	1700	
3·09		3·04	1·71		25·44		Threshold, Wheatsheaf Inn.
					20·69		Cellar, 4 ft. 9 in. below.
	4·83			1·74	23·70	1745	C. road to Lord Nelson.
	4·37		0·46		24·16	1800	
	3·65		0·72		24·88	1900	
6·46		4·29		0·64	24·24	2000	
	6·30		0·16		24·40	2100	
	5·40		0·90		25·30		Threshold, Horse-shoe Inn.
					18·05		Cellar, about middle of Inn, 7 ft. 3 in. below
						2168	S. gable Horse-shoe Inn.
	5·04		0·36		25·66	2186	Opp. C. Church Road.
	4·02		1·02		26·68		Threshold, Grapes Inn.
					24·52		Cellar, 2 ft. 2 in. below.
						2208	In line with S. gable, Grapes Inn.
	4·58			0·56	26·12	2300	
4·04		5·43		0·85	25·27	2400	
	4·70			0·66	24·61	2500	
	4·80			0·10	24·51	2600	
	5·06			0·26	24·25	2681	C. Highfield Lane.
	4·96		0·10		24·35	2800	
		2·31	2·65		27·00		27·00 O.B M. C. of Highfield Lane and Grape Lane.
37·44		35·04	24·79	22·39			

The section line in question was arranged down the centre of the highway, thus forming an irregular line. The section was commenced at an Ordnance bench mark 24·60 opposite the gas works, and ended at an Ordnance bench mark 27·00 at the corner of Highfield Lane and Grape Lane. The first reading was of course a back sight, 2·71, and various intermediate sights were taken before the first foresight which will be seen at 3·21 The level was then

removed to fresh ground, the new back sight being 6·97 (placed in the level book on the same line but of course in a different column from the first fore sight). Intermediate sights were then taken, and the next fore sight was 5·21. The level was then moved and the new back sight was 4·67, and so on, the last sight of all, 2·31, being a fore sight, the staff being held on the Ordnance bench mark at the corner of Highfield Lane and Grape Lane.

The depths of the cellars mentioned were measured with the tape from the top of the ground floor of the respective premises.

The accuracy of the levelling was determined from the fact that the difference between the sum of the back sights, 37·44, and that of the fore sights, 35·04, or 2·40, equalled the difference between the level of the two bench marks, namely, 27·00 – 24·60 or 2·40.

Reducing the Levels.

The reducing of the levels, not having previously been fully explained, is here presented in some detail. After ascertaining that the levels are correct as previously indicated, the rise and fall columns are filled in. The levels are read obliquely from left to right, and referring to the record we have the first sight 2·71 and the next 7·73. This gives 5·02 more staff and consequently a fall as already explained. These figures 5·02 are entered in the fall column in line with the last sight taken into consideration, viz., 7·73. The next sight 7·10 is now dealt with, and is considered with reference to the last sight 7·73. Here we have in the new sight less staff and consequently a rise of 7·73 – 7·10 or 0·63, which is duly entered. The next sight 8·45, considered with 7·10, gives a fall of 1·35.

These operations are continued till we come to the intermediate sight 4·05, taken just before the fore sight 3·21, after which the level is removed to fresh ground. The levels being reduced from left to right, this gives a rise of 4·05 – 3·21, or 0·84, which is set down in the rise column.

The next levels that are considered are those of 6·97 back and 5·92 intermediate, the back sight being on the same sight point as the fore sight 3·21, but being now read from the new line of collimation. Reducing from left to right we have a rise of 6·97 – 5·92 equal to 1·05. The sights 5·92 and 5·32 are next considered, and so on.

As mentioned in the description of the level book, the levels should be so arranged that the last reading on each page is a fore sight. If this is done, the difference between the sum of the rise and fall columns on each page will equal the difference between the sum of the back sights and fore sights, so that each page can be balanced before proceeding further. The sum of the back sights and fore sights on each page is carried forward to the end of the levelling operations in their respective columns, as is also the sum of the rise and fall columns, and, at the completion of the reduction of the levels, the difference of each respective set of columns should be equal.

The reduced levels of the several points are obtained from the rise or the fall entered opposite the location of each point, by adding a rise or deducting a fall to or from the last preceding number in the column of reduced levels, *unless that number has been obtained by means other than a back or fore sight or intermediate sight*, in which case the rise or fall is added to or deducted from the next preceding one obtained by the level. For example:—the reduced levels 23·76, 23·24, 23·38 (p. 121), 20·69, 18·05, 24·52 (p. 122), are those of cellar-floors, taken with the tape or foot-rule, and estimated from the reduced level of the object whence they were measured. Such reduced levels should in the level book be run round with a line or otherwise distinguished, to prevent their being used for calculation of those obtained by observation with the level and staff.

The recorded reduced level of the O. B. M. from which the work commences being 24·60, the fall 5·02 subtracted from it gives 19·58 as the reduced level of the first point noted. The next, a rise of 0·63, added to 19·58 gives 20·21 as the reduced level of the second point; and so on. The reduced level 25·82 of the point at 600 feet is obtained by subtracting its fall, 0·19, not from 23·76 but from the next preceding one 26·01, as above explained.

At the foot of each page the difference of the last reduced level and the reduced level at the very beginning of the section will equal the difference between the back sights and fore sights for the particular page.

The method of plotting the section is shown in Chapter XII.

Cross-sections, to show the surface left and right of certain points in the main section line, are usually taken at right angles to that line, and are set out with the optical square. If they are required to make an oblique angle with the main line (as is often the case at the crossing of roads, especially if these have to be raised or lowered), they are set out with the prismatic compass or the box sextant. Right-angled working cross-sections for a railway whose centre line is already chain-stumped, can be correctly and very expeditiously set out with a stout cord (preferably of sash-line, as it does not kink), 62 yards long, having a loop at each end and a mark at the middle of its length. A ranging-pole is pitched at the chain-stump through which the cross-section is to be taken, the loops are secured at the next chain-stump on each side, the cord is drawn taut at its marked centre, and a pole pitched there. On the other side of the main line a third pole is ranged with these two. This method is specially useful where cross-sections occur on a curve of the main line.

Time and trouble are saved, by setting out the lines of cross-sections in advance of the levelling party, and (if circumstances permit) leaving at each point on the main line a slip of paper containing the distinguishing number of the cross-section, and a memorandum of the distances ("left" and "right" from the main line) at which levels are desired. The staff-holder sets the levelling-staff at these points in succession, calling out the number of the cross-section and the distance of the particular place where he is, as :—"Cross-section No. 2, 20 feet left" [of the main line].

In using the terms "left" and "right," one is understood to be looking forward from the beginning of the main line.

Collimation Method of Reducing Levels.—Another method of laying out levels is known as the collimation method. This, though not in general use, is stated to be easier, and it saves one column of figures. The following shows some of the records of the level book above with the levels reduced according to this method :—

Level Book, with Levels reduced by Collimation.

Back Sights.	Inter-mediates.	Fore Sights.	Height of Line of Collimation.	Reduced Levels.	Chain.	Remarks.
2·71			27·31	24·60		O. B. M. on cottage abutting on brook, and opposite gas works.
	7·73			19·58		Top of 6-in. common tile in footpath just before entrance to brook.
	7·10			20·21		Top of 6-in. earthenware drain.
	8·45			18·86		Invert of 9-in. E.S.P., 89 ft. 3 in. from C. Station Road.
	3·74			23·57		C. Station Road.
	4·85			22·46		Ditto, C. roadway over brook.
	6·82			20·49		Underside of stone bearer, E of brook.
	8·88			18·43		Bottom of brook, E. of road.
	4·05			23·26		C. road. Cannel Leach.
6·97		3·21	31·07	24·10		Ditto.
	5·92			25·15		Ditto.
			And so on.			

Contouring.

Contour lines are lines traversing all points on ground, within a certain area, which are at the same relative level. The process of determining such lines is known as contouring. These lines, when levelled and surveyed, are depicted on a plan, and show the various relative levels of any point.

A plan of any city, town, or district, with the contour lines carefully laid down on it, enables the engineer to devise the best and most economic means of carrying out any system of sewage, drainage, irrigation, or waterworks, as well as such railways or public roads as may be required.

If we imagine a hill to be cut by any number of horizontal planes, and the outline of each cut, as seen from above, to be projected orthographically on the map or plan, the outlines of the cuts so projected are called contour lines. These lines are identical in position with the outlines that would be formed if the sea were to surround the hill and rise to heights corresponding with those at which the horizontal planes just referred to would cut the hill. If a hill were similar in form to a right cone, its contour lines would be represented on the map or plan by a number of concentric circles; the apex of the cone being the centre, and the outermost circle the circumference of the base of the cone. A hill in shape like an oblique cone would be represented by eccentric circles.

The distance on the plan between each contour line is termed the horizontal equivalent.

The Ordnance Survey has done much to render contouring necessary only in extraordinary cases, for on the 6-inch map the contours are at intervals of 25 feet. These intervals are known as vertical intervals. On the parish and town maps also the levels of the principal roads are marked at frequent intervals.

On the appended plate (facing p. 128) is shown a plan, on a scale of ten chains to the inch, of a part of the city of Liverpool, with the contour lines, or lines of equal altitude, represented thereon. The contour lines are shown at every four feet of altitude, as indicated by the numbers inserted on them.

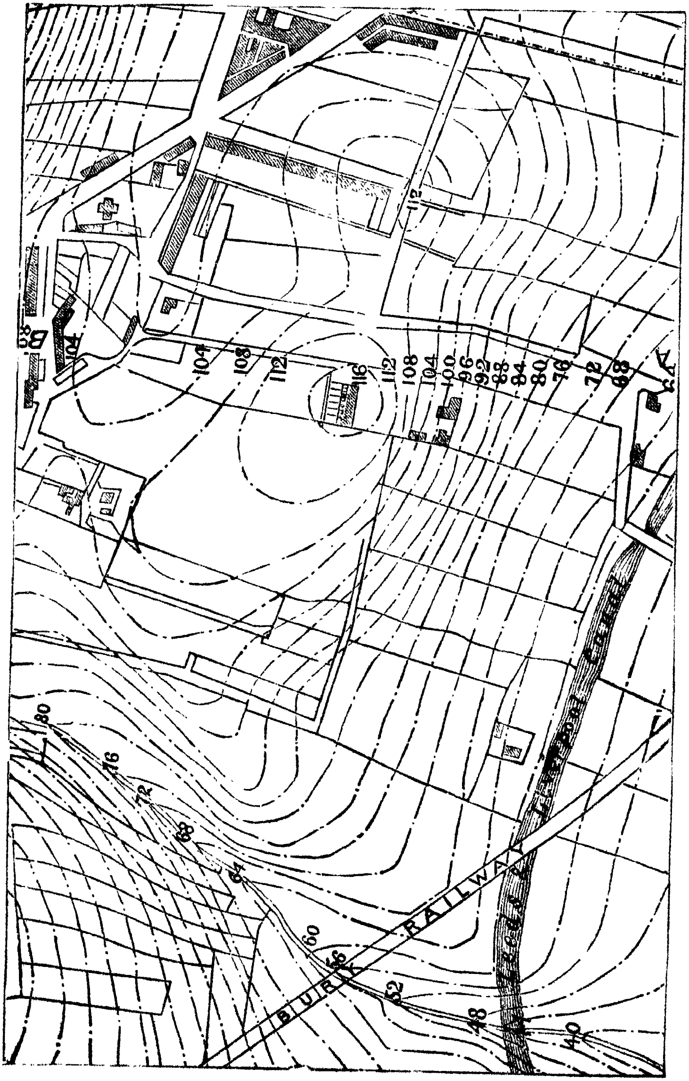
The method generally adopted for determining the position of the contour lines is this: levels are taken along the most suitable streets or roads, or in a direct line between

two or more points of the district to be contoured. The most suitable roads, streets, or lines to be levelled are those which intersect the contour lines at right angles, or nearly so. Bench marks are made in the most favourable places, or stakes are driven in the ground at or near the points where the contour lines will run through; the altitude of each stake being carefully marked on it.

Suppose, for instance, that the line from A to B on the plan had been correctly levelled, and stakes driven down at 64 feet above Ordnance datum, at 68 feet, at 72 feet, and so on. It is evident that the contour lines of corresponding altitudes must pass through the positions of these stakes. Then, in order to determine the 64 feet contour, for example, the level is placed at a distance of 4 or 5 chains from the stake at that altitude, the back staff is placed on the stake, and read off by the level, after it has been carefully levelled; the staff is then sent forward 4 or 5 chains in the direction the contour line seems to take; if the reading of it by the level agrees with what it was on the stake, the position of the staff is in the contour line. If the reading of the staff should be more or less than that at the stake, the staff is to be moved to a higher or lower position, until the reading is the same as that at the stake. When the true position is found, the level is then moved forward, while the staff remains where it is until the back sight or reading is taken, when the staff is again sent forward, and so on. Previously, however, to the staff being sent forward, some distinctive mark should always be placed where it stood, in order that the surveyor may be enabled to survey the contour lines, so as to fix their positions on the map or plan.

The marks usually placed at the positions of the staff on the contour lines are twigs, or small cuttings from the branches of trees. These are stuck in the ground, with pieces of paper at the top, to render them more conspicuous. The surveyor then lays out his chain lines in the most suitable manner, and takes offsets to the twigs or cuttings, and notes them in his field book. He is then able to plot the various courses of the contour lines as if they were fences, and to show, if necessary, the position on the map or plan of every point on the contour line where the staff stood. The dots on the contour lines, on the plan represent the positions of the staff.

Contour Plan of Part of Liverpool.



[*To face page* 128.

If the map or plan be correct, the chain lines for surveying the contours can generally be fixed by the means of the fences and other details.

Levelling with the Theodolite.—(1) The theodolite can be used in the same way as the ordinary dumpy level by clamping the vertical arc, and bringing the bubble of the telescope to the centre of its run, precisely similarly to the preliminary preparations for taking a vertical angle as described in the chapter on angular instruments.

(2) By angular measurement and calculation. The use of the theodolite is sometimes necessary in levelling operations, especially when these operations are required to be conducted over very high and rapidly rising ground, or over steep and almost perpendicular rocks, where the ordinary levelling instrument cannot be fixed.

Select a convenient place to fix the theodolite, where the general inclination of the surface of the country changes, without regarding minor inequalities; then set the instrument level by means of the parallel plate screws, and send an assistant forward with a staff, having a vane or flag fixed to it, of the same height from the ground as the centre of the axis of the telescope of the theodolite. Having gone to the station required, the assistant must hold the vane staff upright, while the observer measures the vertical angle, which an imaginary line connecting the instrument and staff makes, with the horizon. A second observer with a theodolite at the point where the staff is placed, should at the same time with his theodolite, read the angle, which a similar staff placed at the point where the first theodolite is fixed, makes with the horizon. In order that both observations may be made simultaneously a preconcerted signal must be arranged between the observers. The *mean* of the two angles may be considered the correct angle. This precaution is necessary on account of the variableness of the refraction, and more especially so where the points of observation are at a great distance, and one much higher than the other. The distance on the slope must be measured in the meantime, which, with the mean angle, constitute the hypothenuse and angle at the base of a right-angled triangle, in which the base is the horizontal distance between the two stations, and the perpendicular their difference of level.

In this manner, by considering the surface of every prin-

cipal undulation as the hypothenuse of a right-angled triangle, the operation of levelling may be carried on with great rapidity; but without strict accuracy. An error of a single minute produces a difference of level of 3 feet in a distance of two miles. The difference in level is computed from the product of the horizontal distance and the tangent of the mean angle observed.

Levelling with the Barometer.—There are two classes of barometers: (1) the mercurial barometer; (2) the aneroid barometer.

The mercurial barometer is familiar to all, and its use, in the measurement of heights, depends upon the fact that, in a barometer carried to different elevations, the height of the mercurial column, which is balanced by the density and weight of the atmosphere, diminishes as elevation increases.

The height of the column at sea level at a temperature of 32° Fahrenheit is 29·95 inches. Experiment has proved that with an elevation increasing in arithmetical progression, the density of the atmosphere, and consequently the height of the mercurial column balancing it, diminishes in geometrical progression. The difference of reading is nearly 10 feet of increased elevation above sea level for every 0·01 inch decrease in the height of the column.

It is seldom that the mercurial barometer is used, except as a check on another instrument, it having been superseded by the aneroid, as more portable and convenient. Tables of increase in elevation relative to decrease in the height of the mercurial column have been worked out and can easily be procured.

The aneroid barometer is the barometer now almost exclusively used by engineers for approximate determination of altitude. It consists of a chamber corrugated on the outside, from which the air has been exhausted, thus forming a vacuum chamber. The axis of this chamber is connected with a main-spring of very flexible thin steel. A lever arm moving the indicating apparatus, or hand upon the dial, is fixed to the main-spring. The movement of the hand upon the dial depends upon the pressure of the atmosphere upon the vacuum chamber, the surface of which is depressed or elevated as the pressure of the atmosphere is increased or diminished. Upon the dial is marked a scale of altitudes

corresponding to the relative density of the atmosphere at the several elevations.

In levelling with an aneroid barometer another should be kept in a separate place, and its variation recorded at frequent intervals so as to be a check upon the one in use. The temperature should be noted during the time of the observations, and corrections made for it.

Levelling with the Hypsometer.—The hypsometer is a portable instrument, consisting of a thermometer connected with a small boiler, under which is a spirit lamp. Its action depends upon the temperature of the boiling point of water as indicated at relative heights, due to the pressure of the atmosphere. It is well known that water boils when the tension of its vapour is equal to the pressure of the atmosphere, and then the density or weight of the atmosphere diminishes in geometrical progression as the altitude attained increases in arithmetical progression.

Lefroy's rule for indicating altitude by the boiling point of water is as follows:—"Allow for each degree below 212° Fahrenheit that water boils in mean state (or about 30 inches) of barometer, 511 feet for first degree, 513 feet for second degree, 515 feet for third degree, and so on." Hence with the boiling point at 209° Fahrenheit the altitude will be the height at which water boils at 212° + (511 + 513 + 515 feet).

Some observers consider this instrument quite as reliable as the barometer, and there is no doubt that it can usefully serve as a check upon it.

CHAPTER X

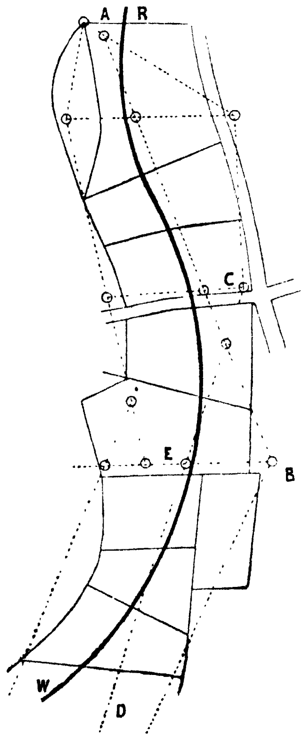
RAILWAY SURVEYING.

PREVIOUS to making surveys of this kind, the line of the proposed railway, as far as it can be determined by trial levels (see Ch. IX., p. 112), must be roughly delineated on a map of the part of the country or district through which it is proposed to pass (Ordnance maps are the best for the purpose), with which the superintendent of the survey must be provided, that he may know what part of the country is required to be surveyed for the intended railway, and in what direction the main survey line ought to be taken, which should be as near the proposed line of railway as the obstructions arising from woods, rivers, &c., will admit.

When, therefore, a new survey is required to be made, range the first base line, fixing stations at the most convenient points, also other stations, to the right and left of the main line, in the direction of fences, roads, rivers, &c. The measurement of the base line, and the filling up of the survey on the right and left may then proceed, as shown in the chapter on Chain Surveying.

When the first base or main line begins to leave the direction of the proposed line of railway, a second main line must then be set out, from a station 10 or 15 chains short of the extremity of the first main line, so that the two main lines may thus be effectually tied together. The survey may be continued similarly with any number of main lines. Sometimes obstructions prevent the effectual tieing together of the main lines, in which case the theodolite must be used, as is shown later.

The width of the survey should be from 5 to 20 or 30 chains; the greater widths being where it has not yet been settled where the line of railway shall pass, or where there are curves or some engineering difficulty.



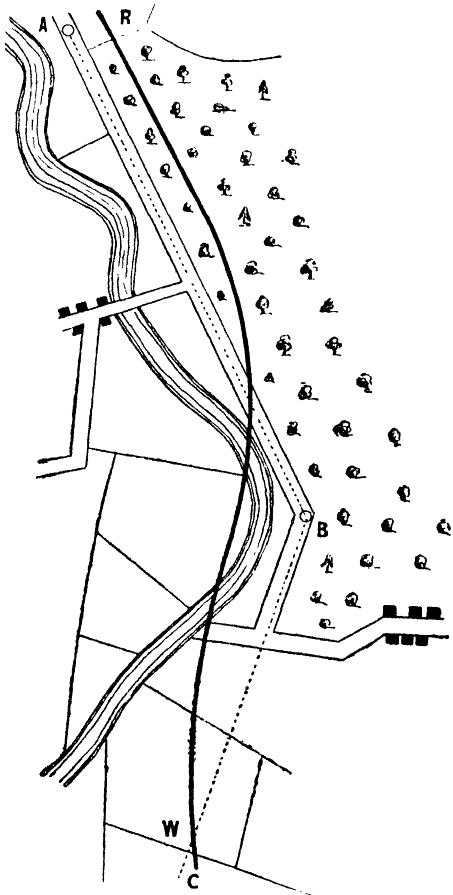
The figure here given represents a survey of this kind. The thick curved line R W is the projected railway; A B the first base line, which at B begins to leave the direction of the line of the railway. At a convenient station in the first main line another main line C D is set out, crossing and recrossing the railway. The line C D is connected with A B by the tie line B E. In the same manner the next main line may be connected with C D, and the survey conducted thus to any extent or in any direction. The filling up of the chief parts of the survey is shown by the dotted lines on both sides of the main lines

Survey of a District for a Railway where Obstructions Occur.

In the next figure (p. 134) R W is a portion of a projected railway, and A B, B C the main lines to survey that portion of the district affected by the railway. The first main line A B runs along a road, till it is obstructed by a large wood on the left, a river being close to its right; and, since the direction of the railway changes near the point of obstruction to the left, a new main line B C is taken.

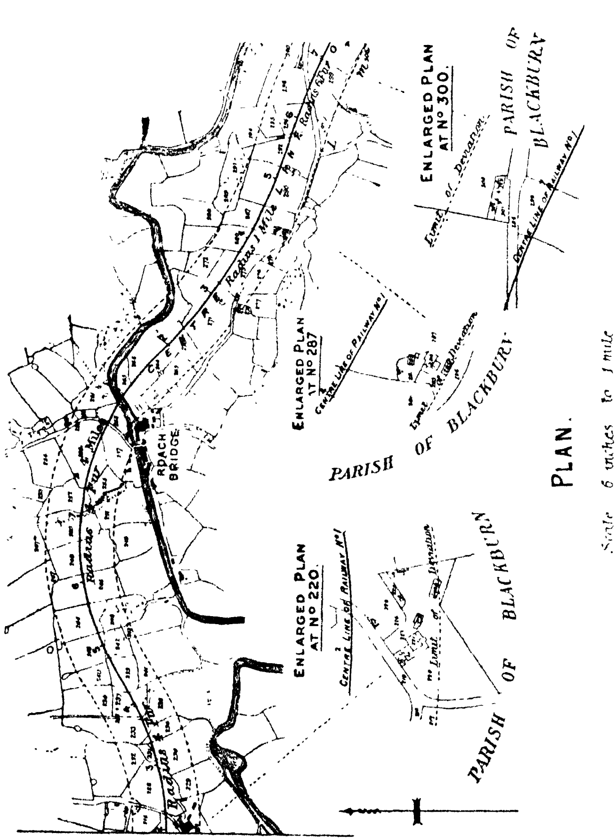
Here the use of the theodolite is indispensable to take the angle of the two main lines A B, B C at B, as it is

impossible, on account of the obstructions, to obtain tie lines to connect the main lines excepting with great trouble. The parts of the survey adjoining the wood and the brook



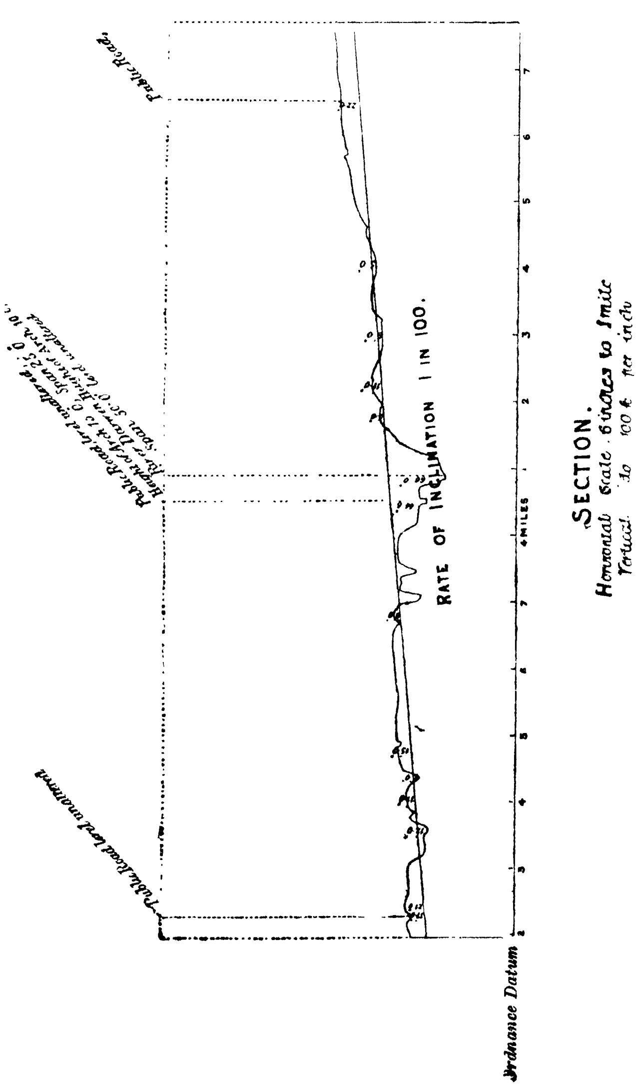
may be filled up in the usual way, while angles must be taken from two or more stations, to determine the positions of objects on the right and left of the railway. As the

railway again leaves the direction of the main line B C at C, a third main line will be required (this line is not shown in the figure); which may be either tied to B C, or its angle of inclination with B C be taken with the theodolite, according to convenience. The filling up of part of the



Reduced Copy of Plan of Line of Railway, as submitted to Parliament

survey to the right and left of the part of B C towards C may be done with the prismatic compass, or box sextant.



Reduced Copy of Section of Line of Railway, as submitted to Parliament.

Railway Plans and Sections.

The illustrations here given of a plan (p. 135) and a section (p. 136) will serve to show the manner in which plans of projected railways are required to be presented to Parliament. Each parcel of land on the plans is numbered, and the plans are accompanied by a "Book of Reference," giving the names of the owner and the lessee or occupier, and a description of every parcel numbered on the plan. Sections and cross-sections are also required.

The limits of deviation, generally shown by thick black dotted lines, usually extend 5 chains on each side of the centre line of the railway, and only within these limits may the promoters vary the line of railway without fresh statutory powers. Sometimes the limits of deviation are placed nearer the central line of railway, to avoid opposition from contemplated expensive severance.

The Standing Orders of the Houses of Parliament affecting Private Bills, to which the reader is referred, prescribe certain requirements which must be rigidly complied with to pass the Examiner on Standing Orders. The scale of the plans must be not less than 4 inches to the mile, and, of large plans required, not less than one quarter of an inch per 100 feet. The sections must be drawn to the same horizontal scale as the plan, and to a vertical scale of not less than 1 inch to each 100 feet. Cross-sections, as required by such Standing Orders, must be on a horizontal scale of not less than 1 inch to every 330 feet, and to a vertical scale of not less than 1 inch to every 40 feet.

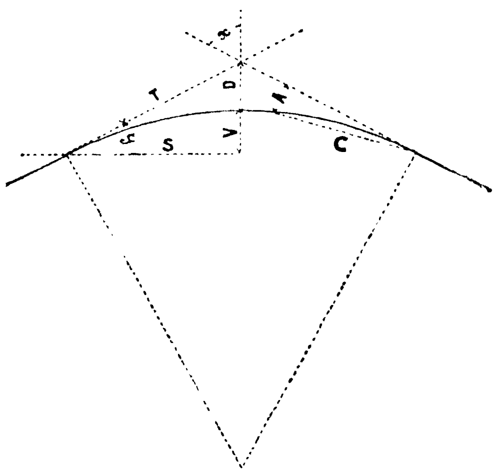
Setting-out Curves.

In railway practice the curve adopted is in almost all cases an arc of a circle, and sometimes two, three, or more consecutive arcs of circles of different radii, having a common tangent or tangents at their point or points of junction, as in the *compound curve.* Frequently the railway curve is composed of two or more circular arcs, having their convexities turned in different directions, with a common tangent or tangents at their point or points of junction : a curve thus composed is called the *serpentine* or *reverse* curve. The straight portions of the railway are always first laid out. and are, in all cases, tangents to the curves at their termini, the curves being in the place of the angles formed by the intersection of the straight portions produced

A length of straight railway always intervenes between two curved lengths. Curves can conveniently be set out by tables, thus saving laborious calculations, and for this reason the radius should be a round number, either a multiple or a sub-multiple of a hundred. Radii are set out either in Gunter's chains, as for instance a curve of 10 chains radius, or in multiples of 100 feet, as a curve of 1,000 feet radius. Given the radius and the angle of intersection of the tangents, the remaining functions of a curve by trigonometry, or in most cases in practice obtained easily, can be worked out from the tables.

Curves of less than 20 chains radius should be set out in ½-chain chords. Curves of more than one mile radius may be set out in 2-chain chords.

The following diagram and formulæ are inserted to render future references intelligible :—



R = Radius of curve.
T = Length of tangent = R tan y.
x = Half angle of intersection.
D = Distance of centre of curve from intersection.
C = Any chord.
A = Tangential angle of C in minutes.
y = Angle of deflection for whole curve.

$$R = \frac{1719\ C}{A} = T \tan x.$$
$$T = R \cot. x$$
$$D = R\ (\text{cosec.}\ x - 1).$$
$$A = \frac{1719\ C.}{R}$$
$$S = R \cos.\ x.$$
$$V = R\ \text{coversine}\ x.$$

$$\text{Number of chords in curve} = \frac{5400 - x.}{A}$$

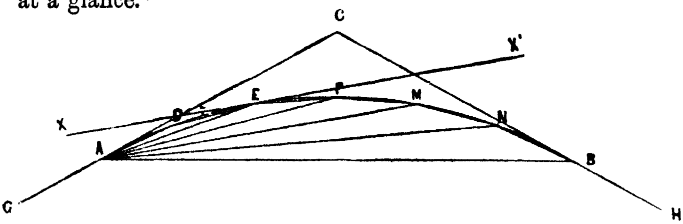
$$\text{Length of curve} = \cdot 000582\ R\ (5400 - x).$$

Note.—x and A in the two preceding formulæ must be expressed in minutes.

There are various systems of ranging curves, the following being illustrated below, viz. :—

(1) With a theodolite and chain.
(2) With two theodolites only.
(3) By offsets in equal chords.
(4) By equidistant points on a curve from two tangents.
(5) By ordinates from chord of whole curve.

Case I.—*To set out a curve with one theodolite and a chain.*— This method is known as Rankine's method of setting out by tangential angles, and, with the method next described, gives the most reliable curves for railways. Mr. Beazeley has published tables on cards for placing on the theodolite, giving the tangential angle required for any common radius at a glance.*



The following data are given: the lengths G A and B H are straight lengths, and A and B the two points to be connected by a curve of 20 chains radius, the pegs to be set

* "Tables of Tangential Angles and Multiples for Setting out Curves from 5 to 200 Radius." London: Crosby Lockwood and Son.

out on the curve 66 feet or 1 chain apart measured as chords. Range G A and B H to intersect in C and drive pegs in the ground at A, C and B. Fix the theodolite at A with its vernier clamped at zero, the telescope being directed on C the point of intersection of the tangents. Now by the formula above, A the tangential angle of any chord in minutes $= \frac{1719\ C}{R}$ therefore in this case $= \frac{1719 \times 1}{20} = 1° \ 26'$.

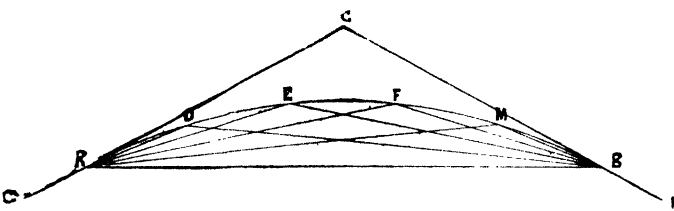
Lay off the tangential angle C A D $= 1° \ 26'$, the chain being extended from the tangent point A to D in the same line as that of the optical axis, or line of collimation of the theodolite. This will give the first point D in the curve. With the theodolite still at A set off the angle C A E $=$ twice the tangential angle $= 2° \ 52'$. Extend the chain from D to meet the line of collimation of the instrument at E, which will be the second point in the curve. The next angle to be set off with the theodolite still remaining at A will be three times the tangential angle or $4° \ 17\frac{2}{3}'$, and, if the chain is extended as before, a third point will be found in the curve, and so on. Pegs must, of course, be driven down at all the points found on the curve.

It may happen that some obstruction, such as a house, &c., may prevent the observer seeing from the point where the theodolite is fixed at A, any further than the point E on the curve. In this case a new tangent must be set out. Remove the theodolite to E. Bring the vernier to zero and clamp the vernier plate, but not the vertical axis. Sight the line E A, clamp the vertical axis, unclamp the vernier plate, and set off the angle X E A $=$ angle C A E. Then the line X E produced to X′ will be the new tangent required. Place pegs at X and X′. To set off the next angle sight on X′ with the vernier clamped at zero. Unclamp the vernier plate and set off the tangential angle X′ E F $= 1° \ 26'$ and extend the chain from E to meet the line of collimation in F. The next angle to be set off, X′ E M, will be double the tangential angle or $2° \ 52'$, and so on. The exact length of the curve may not, however, be an even number of chains, but have a fraction over, say, 80 links $=$ ·8 of a chain. In this case the fraction must be multiplied by the tangential angle, thus $1° \ 26' \times \cdot 8 = 1° \ 9'$.

When the last tangential angle for the even number of chains is set out the next angle will be this plus the angle $1° 9'$.

Case II.—*To set out a curve with two theodolites only.*—This is perhaps the most accurate of all methods of setting out curves, the points in the curve being found by the intersection of the lines of collimation of two theodolites placed at the tangent points A and B respectively. Assume the radius to be 20 chains, and the points in the curve to be 1 chain apart measured as chords.

Now by the formula the number of chords in the curve $= \frac{5400 - x}{A}$. In the illustration it will be seen that there are 5 chords exactly 1 chain in length. Had there been a



fraction of a chord over, the tangential angle for the fraction would be found in the manner indicated in the last example.

By the formula $A = \frac{1719\ C}{R}$ the tangential angle for 1 chord $= 1° 26'$ and for 5 chords $= 7° 9\frac{2}{3}'$. One theodolite is set up at A sighted with the vernier at zero on C, and the first tangential angle C A D $= 1° 26'$ set off, and from the other theodolite being at B with the vernier fixed at zero on C the tangential angle for 5 chains $= 7° 9\frac{2}{3}'$ is set off.

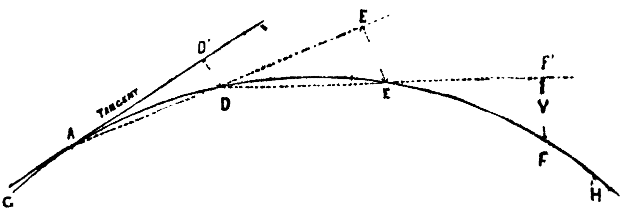
A peg is now placed at the intersection of the lines of collimation of these two instruments, and is the first peg in the curve. The theodolites remaining at A and B the second peg on the curve is set off by the intersection of the line of collimation of the two instruments as before with twice the tangential angle from the instrument at A $= 2° 52'$, and four times the angle for the instrument at B $= 5° 43\frac{2}{3}'$, and so on.

Case III.—*To set out a curve by offsets from equal chords*—This method has been much used, as it does not require the use of an angular instrument, but is open to the objection that if an error be made in setting out any one point in the curve, the error will be repeated in every succeeding one.

Assuming as before the radius to be 20 chains, let L = length measured from tangent-point along the tangent, for first offset-point; and from the several points in the curve, along their chords produced, for succeeding offset-points: and let O = offset from the length so measured. The formula is :—$O = R - \sqrt{R^2 - L^2}$. Succeeding offsets = twice the offset from tangent.

Where L is less than $\frac{R}{20}$, a simpler formula, viz. :—$O = \frac{L^2}{2\,R}$, is sufficiently accurate for practical purposes.

Let G A be a length of straight line and A tangent point at which the curve commences, measure from A in G A



produced A D′ = 1 chain, set off perpendicular to it D D′ = $\frac{66 \times 66}{2 \times 20 \times 66} = \frac{66}{40} = 1 \cdot 65$ foot. Place a peg at D, which will be the first point in the curve. Range a straight line through the points A D produced to E′ making D E′ = 1 chain. As before set off the offset E E′ = twice D D′ = 3·30 feet. E will then be the second point on the curve. D E is then produced to F′, making E F′ = 1 chain. Set off offset F′ F = 3·30 feet as before, and so on to the end of the curve.

Case IV.—*To set out a curve by offsets from the tangents.*—This method consists in setting out equidistant points on the curve by ordinates from two tangents. For accurate work it cannot compare with a curve set out by the theodolite.

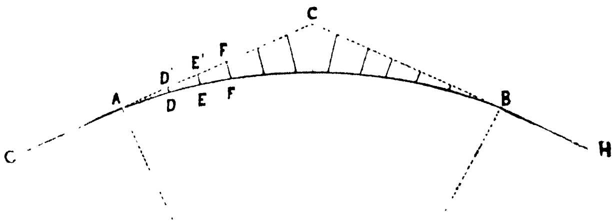
Assume as before the radius to be 20 chains, and the

points set out on the curve 1 chain apart. The same formula is necessary as in Case 3 above, viz. :—

First offset from tangent $= \frac{\text{chord}^2}{2 \text{ radius}}$ thus offset D′ D = 1·65 feet.

The remaining offsets or ordinates as E′ E, F′ F, &c., are found thus :—

Second ordinate or E′ E the second point on the curve $= 2 \times 2 \times 1'{\cdot}65 =$ first ordinate $\times 2^2 = 6{\cdot}60$ feet.

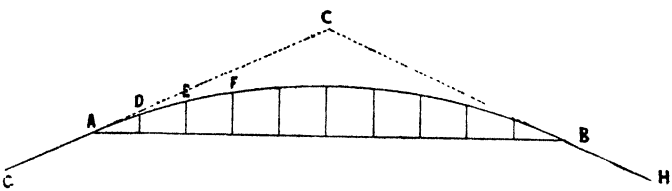


Third ordinate or F′ F the third point on the curve $= 3 \times 3 \times 1{\cdot}65 =$ first ordinate $\times 3^2 = 14{\cdot}85$ feet, and so on.

The number of the point on the curve being squared and multiplied by the first offset will give the offset for the point desired.

The Table of offsets given on the following page will be found useful in reducing to a minimum the calculation required when actually setting out curves on the ground.

Case V.—*To set out a curve by ordinates from the chord of the whole curve.*—This method is suitable for rough purposes, such as curves for roads, &c. The length of the chord must be measured and divided into 10 parts, from which ordinates



AT THE END OF FIRST CHAIN FROM TANGENT-POINT OF CURVES.

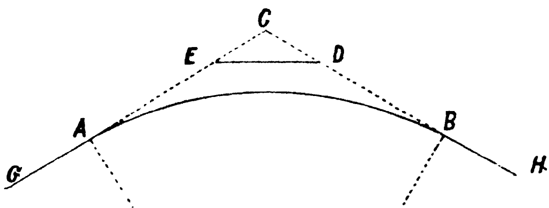
Rad. in chs.	Offsets in ins. and dec.	Rad. in chs.	Offsets in ins. and dec	Rad. in chs	Offsets in ins. and dec	Rad. in chs.	Offsets in ins. and dec	Rad. in chs.	Offsets in ins. and dec.
5	80·01	45	8·80	84	4·71	138	2·87	210	1·89
6	66·46	46	8·61	85	4·66	140	2·83	215	1·84
7	56·86	47	8·43	86	4·60	142	2·79	220	1·80
8	49·69	48	8·25	87	4·55	144	2·75	225	1·76
9	44·14	49	8·08	88	4·50	145	2·73	230	1·72
10	39·70	50	7·92	89	4·45	146	2·71	235	1·69
11	36·07	51	7·77	90	4·40	148	2·68	240	1·65
12	33·06	52	7·62	91	4·35	150	2·64	245	1·62
13	30·51	53	7·47	92	4·30	152	2·61	250	1·58
14	28·32	54	7·33	93	4·26	154	2·57	255	1·55
15	26·43	55	7·20	94	4·21	155	2·55	260	1·52
16	24·77	56	7·07	95	4·17	156	2·54	265	1·49
17	23·31	57	6·95	96	4·13	158	2·51	270	1·47
18	22·02	58	6·83	97	4·08	160	2·48	275	1·44
19	20·86	59	6·71	98	4·04	162	2·44	280	1·41
20	19·81	60	6·60	99	4·00	164	2·41	285	1·39
21	18·87	61	6·49	100	3·96	165	2·40	290	1·37
22	18·01	62	6·39	102	3·89	166	2·39	295	1·34
23	17·23	63	6·29	104	3·82	168	2·36	300	1·32
24	16·51	64	6·19	105	3·77	170	2·33	305	1·30
25	15·85	65	6·09	106	3·75	172	2·30	310	1·28
26	15·24	66	6·00	108	3·66	174	2·28	315	1·26
27	14·67	67	5·91	110	3·59	175	2·26	320	1·24
28	14·15	68	5·82	112	3·54	176	2·25	325	1·22
29	13·66	69	5·74	114	3·47	178	2·22	330	1·20
30	13·20	70	5·66	115	3·44	180	2·20	335	1·18
31	12·78	71	5·58	116	3·41	182	2·18	340	1·16
32	12·38	72	5·50	118	3·36	184	2·15	345	1·15
33	12·00	73	5·42	120	3·30	185	2·14	350	1·13
34	11·65	74	5·35	122	3·25	186	2·13	355	1·12
35	11·32	75	5·28	124	3·19	188	2·11	360	1·10
36	11·00	76	5·21	125	3·17	190	2·08	365	1·08
37	10·70	77	5·14	126	3·14	192	2·06	370	1·07
38	10·42	78	5·08	128	3·09	194	2·04	375	1·06
39	10·16	79	5·01	130	3·05	195	2·03	380	1·04
40	9·90	80	4·95	132	3·00	196	2·02	385	1·03
41	9·66	81	4·89	134	2·96	198	2·00	390	1·02
42	9·43	82	4·83	135	2·93	200	1·98	395	1·00
43	9·21	83	4·77	136	2·91	205	1·93	400	0·99
44	9·00								

may be set out to give points on the curve. The formula required is :—

$$\text{Length of any ordinate } O = \sqrt{R^2 - X^2} - (R - V)$$

When R = radius of curve in feet.
X = distance of ordinate from centre of chord.
V = versed sine or height of ordinate at centre of curve.*

Case VI.—*To find the position of the point of intersection of the tangents when it is inaccessible.*—When the point of intersection of the tangents happens to fall in water the following method is resorted to for finding the inaccessible lengths.



Produce the tangents G A and H B to E and D respectively. Measure E D, and observe angles D E A and E D B. Then angle C E D = 180° – angle A E D and angle C D E = 180° – angle E D B and angle A C B = 180° – (C E D + C D E).

$$\text{And } CE = \frac{ED \sin. CDE}{\sin. ACB}$$

$$\text{And } CD = \frac{ED \sin. CED}{\sin. ACB}$$

Case VII.—*To find the length of a curve, and to find the number of chords by the formula given on page* 139.

$$\text{Number of chords on the curve} = \frac{5400 - x}{A}$$

$$\text{Length of curve} = {\cdot}000582\ R\ (5400 - x)$$

* Molesworth's "Pocket-Book of Engineering Formulæ."

Setting out the Surface Widths of Railways.

After the centre of a railway has been marked out, the line must again be carefully levelled, cross-sections taken, and the pegs that mark the line numbered and entered consecutively in the level book in a vertical column, with the corresponding depths of cuttings or embankments in a second column (see form of Level Book, p. 152). These depths are estimated from the formation level, which is commonly about 2 feet below the intended line of the rails; the 2 feet are afterwards to be filled up to form the permanent way.

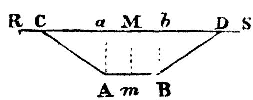
The line is now prepared for setting out the surface widths, the simplest case of which is when the surface of the ground is level, as well as coincident with the formation level of the intended railway. In this case it is only required to set out half the width of the formation level on each side of the centre peg perpendicular to the direction of the railway, adding to each half width the intended width of the side fence, and putting down pegs to mark the half widths and breadths of the fences.

When the surface of the ground is above or below the formation level, as is commonly the case, the widths must be set out as in the subjoined Cases I. to IV.

Two sets of pegs must always be used, the one for distances, the other for levels, differently painted or otherwise distinguished. Pegs are generally set out at certain distances from the true position of the stake in a cutting, which is of course removed in the work.

Case I.—*To set out the width of a railway cutting, when the surface of the ground is laterally level, and at a given height above the formation level, the ratio of the slopes* being given.*

In the annexed cross section of the cutting, R S is the horizontal surface of the ground; A B the formation level; A C, B D the side slopes; M the middle stake, and M m = A a = B b the perpendicular depth of the cutting. Multiply the depth M m by the ratio of the slopes, to which add the half width A m or a M, the sum is half the surface width to be set out from M to C; after which set out R C



* The ratio of the slopes is the proportion that the distance C a bears to the depth A a. When this ratio is as 1 to 1, C a = A a; when

for the width of the fence. The same operation must be repeated on the other side of M.

Example.—Let the width of the formation level A B = 33 feet, the depth M *m* of the cutting 30, the ratio of the slopes as $1\frac{1}{2}$ to 1, and the width of the side fences each 6 feet; required the width of the cutting, and the width of land included by the fences.

$$30 \times 1\tfrac{1}{2} + \tfrac{33}{2} = 45 + 16\tfrac{1}{2} = 61\tfrac{1}{2} \text{ feet} = MC = MD = \tfrac{1}{2} \text{ width of cutting; and } 61\tfrac{1}{2} + 6 = 67\tfrac{1}{2} = MR = MS = \tfrac{1}{2} \text{ width of land.}$$

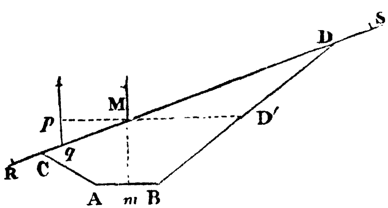
The double of this is the whole width.

The numbers in the column marked "computed half-widths" in the level book are found as herein.

If the figure A B D C be inverted, it will represent the cross section of an embankment; for setting out the width of which the same method obviously applies.

Case II.—*The same data being given as in the last case, to set out the width of the cutting, when the ground is laterally sloping, the lateral fall of the ground in a given horizontal distance being also given.*

Let C D be the sloping surface of the ground, A B D C the cross section of the cutting, and p D′ a horizontal line passing through the centre stake M, M D′ being the computed half width of the cutting—Fix the level so that by turning the telescope 2, 3 or more chains of the line may be seen, on both sides of it; set up a levelling staff at M, and another at q, not exceeding a chain's distance from M, observing the level readings on both staves, the difference of which is equal to $p\,q$; measure the sloping and horizontal distances M q, M p with a tape in feet. Then take the computed half width M D′ (found as in Case I.), and



it is as $1\frac{1}{2}$ to 1, C a = $1\frac{1}{2}$ times A a. This ratio varies according to the nature of the material through which the cutting is made, being less in rocky or clay ground, and greater in soft or sandy ground.

multiply it by M q, reserving the product; multiply the difference of the stave readings $p\,q$ by the ratio of the slopes, add and subtract the product to and from the horizontal distance M p, reserving the sum and difference; lastly, the reserved product, being divided by the reserved sum, will give the corrected half width M C, and by the reserved difference the corrected half width M D.

Example.—The depth of the cutting at M is 22 feet, the bottom width A B = 36, the sloping distance M q = 25, the level distance M p = 24, the difference of the readings of the staves $p\,q$ = 7 feet, and the ratio of the slopes as $1\frac{1}{2}$ to 1. What are the corrected half widths M C and M D?

$$22 \times 1\tfrac{1}{2} + \tfrac{36}{2} = 33 + 18 = 51 \text{ feet} = \text{computed half width.}$$
$$\begin{array}{r}25\\ \hline 1275 \text{ reserved product}\end{array}$$

$$\begin{array}{rl} & 24 \\ 7 \times 1\frac{1}{2} = & 10\frac{1}{2} \\ \hline \text{reserved sum} = & 34\frac{1}{2})\ 1275\ (36{\cdot}95 \text{ feet} = \text{cor. } \tfrac{1}{2} \text{ width M C.} \\ \hline \text{reserved diff.} = & 13\frac{1}{2})\ 1275\ (94{\cdot}44 \text{ feet} = \text{cor. } \tfrac{1}{2} \text{ width M D.} \end{array}$$

The following general formula for the values of M C M D, is easily remembered:—

$$\text{Corrected } \tfrac{1}{2} \text{ width} = \frac{\omega\, s}{l \pm r\,h}$$

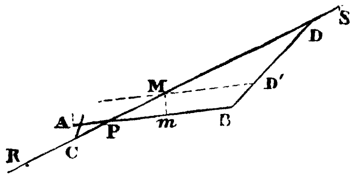
The positive sign is used for M C, and the negative for M D; M D′ being $= \omega'$, $p\,q = h$, M $q = s$, M $p = l$, and the ratio of the slopes as r : 1.

If the figure A B D C be inverted, it will obviously represent the cross section of an embankment of like dimensions, the longer distance, in this case, being measured down, and the shorter up the slope.

Case III.—*The same data being given as in Case II., to set out the width, when it consists partly of a cutting and partly of an embankment.*

In the annexed figure B D P C A is the cross section of

the works of a railway, consisting of the cutting B P D and the embankment A P C; A B is the formation level, M the central stake, and D C the sloping surface of the ground. When the cutting B P D extends over more than half the formation level A B, the corrected half width M D is found as in Case II. But to find the other corrected half width M C, passing under the embankment A P C, proceed as follows. Multiply M D by the difference of the width of the formation level A B and the estimated half width M D′, and divide the product by M D′, and the result will be the corrected half width M C.



Example.—Let the bottom width A B = 36 feet, the depth M m = 4 feet, the ratio of the slopes as 2 : 1, and the difference of level readings 7 feet at 25 and 24 feet from the central stake, respectively measured on the slope and horizontally; required the corrected half width M D, M C.

By Case I. . $4 \times 2 + \frac{36}{2} = 26$ feet = estimated ½ width M D′

By Case II. . $\frac{26 \times 25}{24 - (7 \times 2)} = 65$ feet = corrected ½ width M D.

By Case III. $\frac{65 \times (36 - 26)}{26} = 25$ feet = corrected ½ width M C.

The formula for finding the corrected half-width, where there is both a cutting and an embankment, is thus—

$$\text{M C} = \frac{(\omega - \acute{\omega})\, s}{l - r\, h},$$

wherein ω is the width of formation level, and the other symbols the same as in the previous cases.

To find the distance from M to P, where the cutting and embankment meet, use the following proportion—

as $p\,q$: M q : : M m : M P,

that is, $7 : 25 :: 4 : \frac{25 \times 4}{7} = 14{\cdot}28$ feet = M P.

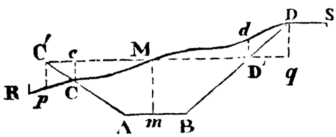
When the sloping surface of the cutting passes through the middle point of the formation level, that is, when the points M m and P coincide; then $\omega' = \frac{1}{2}\omega$, and the formula $MC = \frac{(\omega - \omega')s}{l - rh}$ (p. 149)

$$\text{becomes } MC = MD = \frac{\frac{1}{2}\omega s}{l - rh}.$$

In this case the cutting and embankment are equal.

By inverting the cross section of this problem it will be seen that a like calculation will be required, in the case of A P C being a cutting, and B P D an embankment.

Case IV.—*To find the surface width of a cutting where the ground is very uneven.*



The annexed figure is a cross section of a cutting, wherein the surface C M D of the ground is very uneven. The method of finding the surface width is by approximation, and will be best shown by an example in numbers.

Let A B = 36 feet, M m = 32, and the ratio of the slopes as 2 : 1; then M C′ = M D′ = (32 × 2) + ($\frac{1}{2}$ × 36) = 82 feet; lay off this distance horizontally from M to d; then d is directly above D′. Observe the difference of level readings at M and d, which in this case is 9 feet; which being multiplied by the ratio of the slopes, that is by 2, gives 18 feet = approximate distance D d; whence M D = M d + d D = 82 + 18 = 100 feet. Again, place a level staff at D, and the reading will be found to be 9·7 feet greater than that at M, or 9·7 − 9 = 0·7 feet greater than at d; therefore the place of the point D requires further correction, which is thus effected; 0·7 × 2 = 1·4 feet = second correction; whence M D = 100 + 1·4 = 101·4 feet; which as the latter correction is small, may be safely assumed to be the true distance of D from M, or the horizontal distance M q. The method of finding the other corrected half-width M C = horizontal distance M c, is the same as that just given, excepting that the repeated corrections are subtracted from the computed half-width instead

of being added thereto. In this manner the horizontal distance of C from M is found to be 61·8 feet.

When the differences of the levels at M, d and D are great, it will require three, four, or more approximations, similar to those just given, to each of the corrected half-widths.

This cross section may be inverted for an embankment, as in the preceding cases.

Case V.—*To calculate the quantity of land for a projected railway.*

In preparing the estimates for a projected railway, the required quantity of land is commonly found, without respect to the lateral sloping of the surface of the ground, by taking considerable lengths of regularly rising or falling ground in one calculation, the depths of the ends of such lengths being measured for the purpose with the vertical scale.

RULE.—Find the surface widths, fences included, from the given depths, at each end of the given length, by Case I., multiply their sum by the length in chains, and divide the product by 1320* for the product in acres.

Example.—Let the length be 16 chains, the depth of the cross sections at the ends 18 and 58 feet, the width of the formation level 36 feet, the ratio of the slopes as 2 to 1, and the width of the fences 6 feet each; required the area of the surface. By Case I.—

$$36 + (18 \times 4) + (6 \times 2) = 120 = \text{width of one end.}$$
$$36 + (58 \times 4) + (6 \times 2) = 280 = \text{width of the other.}$$

$$400 = \text{sum of widths.}$$
$$16$$

$$1320)6400$$

$$\text{The content} = 4{\cdot}84848 = 4a.\ 3r.\ 16p.$$

* The divisor 1320 is thus obtained:—The area of a trapezoid being = half the sum of the two parallel sides multiplied by the perpendicular distance between them; their sum (400 feet in the example), before being multiplied by the length (16 chains in the ex-

It is usual in practice to find the actual contents of the ground required from the several proprietors for the works of a railway, by measurements from the plan.

Level Book for Railway Surveys.

A level book of the following form is used in setting out the cuttings and embankments of railways :—

	No. of Stake.	Depths of Cuttings or Embankments.	Computed Half-Widths.	Corrected Half-Widths to Edge of Cuttings or Foot of Embankments. Left.	Right.	Whole Widths including Fences, each 6 feet in width.
		feet	feet.	feet.	feet.	feet.
cutting	183	28·00	57·00	61·80	101·40	175·20
cutting	184	4·00	21·00	22·63	48·37	83·00
cutting	185	30·60	60·90	64·77	71·32	148·09
embankment.	186	2·16	18·24	18·00	19·44	49·44
embankment.	187	16·08	39·12	40·68	58·24	110·92
embankment.	188	20·00	30·00	36·16	48·00	96·16

NOTE.—The depths in the second column are found by calculation, or by careful measurement from the sections; but the latter method is the less correct of the two. The computed half-widths, in the third column, are found by Case I. ; the corrected half-widths, in the fourth and fifth column, by Cases II., III., and IV., and the widths in the last column are the sums of those in the fourth and fifth *plus* the breadths of the two side fences of the railway.

ample), would in the ordinary course be first divided by 2. The product (6400 in the text) is in terms of chain-lengths 1 foot wide ; and to reduce this to terms of square chains it must be divided by 66. There being 10 square chains in an acre, the resulting quotient must be divided by 10. Therefore, when the sum of the widths in feet is multiplied by the length in chains, the product has to be divided by $2 \times 66 \times 10 = 1320$, to obtain the area in acres.

CHAPTER XI.

SUNDRY CASES ARISING IN THE LAYING OUT AND DIVISION OF LAND.

THE simpler cases arising in the laying out and division of land have already been dealt with in Chapter I., and the following cases are intended for the further guidance and assistance of the student in the same direction.

Case I.—*To lay out a given quantity of land in the form of a rectangle, the length of which shall have a given proportion to its breadth: that is, the length shall be to the breadth as 3 to 1, or as 4 to 3, or as 5 to 2, &c.*

RULE.—Divide the given area by the product of the two terms of the proportion, and the square root of the quotient, multiplied separately by the terms of the proportion, will give the required length and breadth.

Examples.—1. Lay out 3 acres of land in the form of a rectangle, the length of which shall be to its breadth as 3 to 2.

$3 \times 2 = 6$)300000 square links in 3 acres

```
       50000(223·6
       4
     ------
   42)100
       84
     ------
  443)1600
      1329
     ------
 4466)27100
      26796
     ------
        304
```

links.
$223{\cdot}6 \times 3 = 670{\cdot}8$ length.
$223{\cdot}6 \times 2 = 447{\cdot}2$ breadth.

2. Lay out $6\frac{1}{4}$ acres in a rectangular form, the ratio of whose length and breadth shall be as 5 : 2.

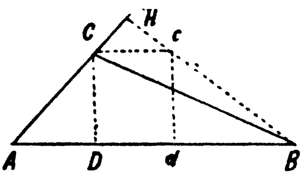
$5 \times 2 = 10)625000$ links.

$\sqrt{62500} = 250.$ $250 \times 5 = 1250 =$ length. $250 \times 2 = 500 =$ breadth.

Case II.—*To lay out a triangle, of a given area and a given base, one of the sides of which shall have a given position.*

RULE 1.—Divide twice the area by the base for the perpendicular, which erect at any point in the base, and at the extremity of the perpendicular and at right angles to it, range a line till it meet the side given in position, which point of meeting is the vertex of the triangle required.

Examples.—1. It is required to lay out a triangle A B C to contain 2*a*. 2*r*. 16*p*. on a base A B of 1200 links, the position of A H, of which the side A C is a part, being given,



By Rule 1

$2a.\ 2r.\ 16p. = 260000$ links

$\times 2$

$1200)520000$

$433\frac{1}{3}$ links $=$ C D

On any point d of the given base A B = 1200 links, erect the perpendicular $dc = 433\frac{1}{3}$ links ; range c C perpendicular to dc till it meet A H in C; then B C being joined, A B C is the triangle required. For let C D be drawn perpendicular to A B, then, by the nature of the construction, C D = $c\,d$. Q. E. D.

RULE 2.—Measure H B perpendicular to A H, the line given in position, to the end B of the given base, and divide twice the given area by H B, and the quotient is A C, which, being measured off, will give the point C.

2. Solve the last example by Rule 2, the perpendicular B H being 860 links.

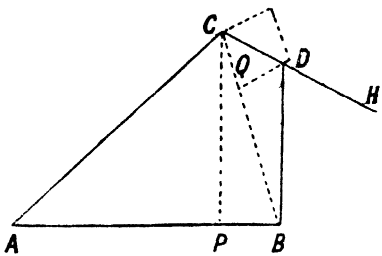
$$\frac{260000 \times 2}{860} = 604{\cdot}65 \text{ links} = \text{A C}.$$

Case III.—*To lay out a trapezium of given area, the positions and lengths of two of its adjoining sides being given, and also the position of its third side.*

RULE.—Join the ends of the two adjacent sides, thus

forming a triangle, the area of which must be found, which area must be subtracted from the given area, and the remaining area must be laid out in the form of a triangle, on the line joining the two given sides, as base, by Case II.

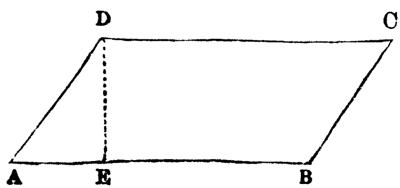
Example.—Let the given area of the trapezium A B D C be 6 acres, A B, A C the sides that are given in length and position, and C H the position of the third side. Join B C; measure the perpendicular C P, which is found = 840 links, and C B = 880 links, A B being = 1200 links; whence the area of the triangle A B C = 600 × 840 = 504000 square links; and 600000 − 504000 = 96000 = area of the triangle B C D; therefore 96000 × 2 ÷ 880 = $218\frac{2}{11}$ links = perpendicular D Q; which, it will be seen, is applied as in Case II.; thus constituting the required trapezium A B D C. C D may also be found as by Rule 2, Case II.



Case IV.—*To lay out a rhomboid oj given area, the lengths of its sides being given.*

Rule.—Divide the area by the longer side, subtract the square of the quotient from the square of the shorter side, and the square root of the remainder is the distance of the perpendicular from the end of the larger side, which perpendicular, being made equal to the quotient first found, gives the breadth of the rhomboid.

Examples.—1. The area of a rhomboid is required to be 6 acres, and its longer and shorter sides to be respectively 1200 and 625 links, required the place and length of its perpendicular.



$$1200)600000$$

$$500 = \text{perpr.} = \text{D E}$$

$$625^2 = 390625$$
$$500^2 = 250000$$

$$\sqrt{140625} = 375 \text{ links} = \text{A E}.$$

2. Lay out the figure when its sides are each 4 chains, that is, when it is a rhombus, its given area being 1*a*. 1*r*. 1*p*.

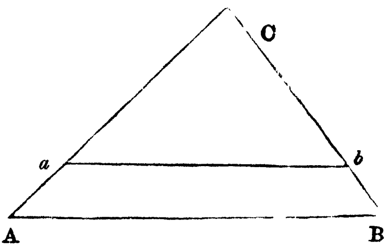
Ans. 314·06 links = perpendicular, and its distance 247·72 links.

Case V.—*To divide any proposed quantity of land from a triangle, by a line parallel to one of its sides.*

RULE.—Since the areas of triangles are as the squares of their like or homologous sides, we shall have

Area Δ A B C : area Δ *a b* C : : A C² : *a* C².

See Theor. X., page 6.



Examples.—1. Let the base A B = 2600 links, A C = 2000, and B C = 1600, cut off 6*a*. 1*r*. 24*p*. by the line *a b* parallel to A B.

From the three sides the area of the triangle A B C is found = 1599218 square links,
and 6*a*. 1*r*. 24*p*. = 640000 square links,
the difference = 959218 = area of Δ *a b* C.

Then per Rule, 1599218 : 959218 : : 2000² : 2399218; and $\sqrt{2399218}$ = 1548·94 links = *a* C; whence 2000 − 1548·94 = 451·06 links = A *a*. Again by similar triangles 2000 : 1600 : : 451·06 : 360·85 links = B *b*. Therefore, measure on the ground the distances A *a*, B *b*, just found, respectively, from the points A and B; and range the line *a b*, which will divide the given quantity A B *b a* from the triangular field A B C, as required.

NOTE.—In this manner a triangle may be divided into any number of equal or unequal parts, by lines parallel to any

one of its sides, by successively subtracting the sums of 2, 3, 4, &c., areas from the area of the triangle, and making the remainders successively the second terms of the proportion.

2. The sides of a triangular field are 900, 750, and 600 links ; it is required to cut $0a.\ 3r.\ 28p.$ therefrom, by a straight fence parallel to its least side.

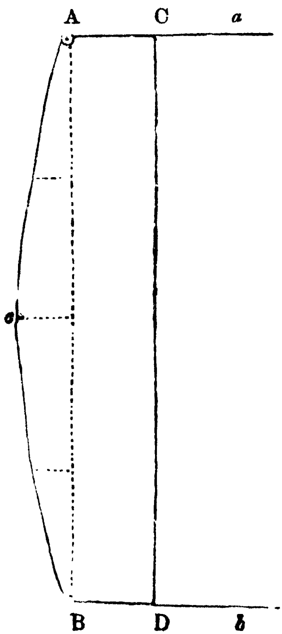
Ans. The distance from the ends of the least side, on the largest and intermediate sides, are respectively $211\frac{1}{4}$ and 176 links.

Case VI.—*To lay off any given quantity of land from an irregular field.*

RULE.—First find the area of the crooked or irregular part by means of offsets, which area must be deducted from the given area, after which the remaining area must be laid out by means of the Cases above cited.

A.—When one side of the field is crooked and two straight and parallel.

Example.—Let a A c B b be a portion of a field or common, the side A c B being crooked and A a, B b parallel, the length of A B is 2400 links, and the area of the offset piece A c B is found to be 227500 square links ; it is required to cut off 12 acres from the field by a line parallel to A B.



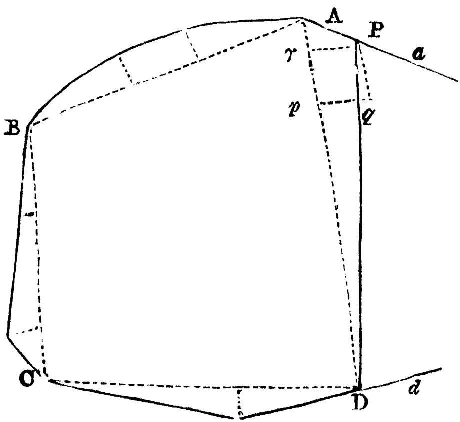
$$\begin{array}{rl}
\text{12 acres} = & 1200000 \text{ sq. links,} \\
\text{offset piece} = & 227500 \text{ ditto} \\
\hline
24{\cdot}00 \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. & \begin{array}{l} 2)9725{\cdot}00 \\ \hline 12)4862{\cdot}5 \\ \hline \end{array} \\
& 405{\cdot}2 = AC = BD
\end{array}$$

Therefore, measure the distances A C = B D = $405\frac{1}{6}$ links on the ground, and range the straight line C D, then shall the space A c B D C contain 12 acres.

B.—When all the sides of the field or common are crooked.

Example.—Let a A B C D d be a portion of a field or common, the boundary of which is crooked, it is required to part therefrom 6 acres, by a straight fence running from D towards A.

Measure the four lines AB, BC, CD, DA, taking the offsets on the three first, and finding the area of the trapezium



A B C D in the usual way, which united areas are found to be 570000 square links, the line A D being 810 links.

$$
\begin{array}{rl}
\text{Hence 6 acres} = & 600000 \text{ square links} \\
\text{Area A B C D and offsets} = & 570000 \text{ ditto} \\
\hline
81{\cdot}0 \left\{ \begin{array}{l} 9)3000{\cdot}0 \\ \hline 9)333{\cdot}3 \end{array} \right. & \\
\hline
& 37{\cdot}037 \\
& \quad\quad 2 \\
\hline
& 74{\cdot}074 \text{ links} = \text{perpend. P}\,r.
\end{array}
$$

Therefore, apply the perpendicular P r = 74·074 links as in Case II., and range the straight line A P; then shall the 6 acres be parted from the field as required,

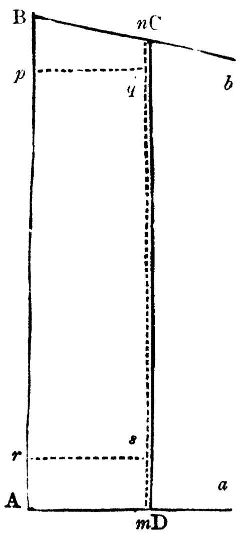
Case VII.—*To divide from a field, bounded by straight fences, any given quantity of land, by a line parallel to one of the straight fences.*

RULE.—Measure the length of the straight fence or line to which the division line is required to be parallel; divide the given quantity by that length, and set the quotient perpendicular to the given straight fence, at two points near the ends thereof, ranging a line through the extremities of the two perpendiculars, and measuring the length thereof; find the area of the trapezoid thus obtained, and take the difference between this area and the given area, which difference, being divided by the last measured line, will give the breadth to be set out perpendicularly from the last measured line, either inwardly or outwardly, accordingly as the difference is in excess or decrease of the given quantity.

Example.—Let *a* A B *b* be a portion of a field or common with straight fences, it is required to lay out 10*a.* 2*r.* 16*p.* by a line parallel to A B, the length of which is 2200 links.

$$2200\begin{cases} 2)1060000 = 10a.\ 2r.\ 16p. \\ \overline{11)5300} \end{cases}$$

$$\overline{482} \text{ nearly} = p\,q = r\,s.$$



Now the perpendiculars *p q*, *r s* being set off, and the line *m n* measured, is found to be 2025 links: whence

$$(2200 + 2025) \times \frac{482}{2} = 1018225 \text{ sq. links, and}$$
$$(1060000 - 1018225) \div 2025 = 20{\cdot}6 \text{ links,}$$

the distance to be set perpendicularly at both ends of *m n*, to the right thereof, because the area cut off by *m n* is less than the given quantity, and the straight line C D ranged through the extremities of these perpendiculars will be parallel to A B, and will cut off the required quantity.

NOTE 1.—Those who are accustomed to operations in analytical trigonometry, will find the following formulæ much more expeditious for laying out land by means of parallel fences, as in Case VII., especially where a great number of successive inclosures are required to be laid out:—Let $A B = a$, $\sigma =$ sine $\angle A$ $s =$ sine $\angle B$, $S =$ sine $(\sigma + s)$, and $P =$ area A B C D, which is required to be cut off; then,

$$B C = \frac{1}{S}\left(a\,\sigma - \sqrt{\frac{\sigma}{s}\,(a^2 s\,\sigma - 2 P S)}\right),$$

and $A D = \frac{s}{\sigma} B C.$

Whence both B C and A D become known without any preliminary measurement, except that of the side A B and the angles A and B: and by adding the next area to be cut off to P, and calling the sum Q, and then substituting Q for P in the formulæ for B C, the next following distances B b and A a may be found; and so on for any number of inclosures.

When A and B are supplemental angles, the lines A a, B b, will be parallel, which makes the problem extremely simple; and when the sum of the angles at A and B are greater than two right angles, S becomes negative, and the lines A a, B b, will meet on the other side of A B. In this case the formulæ will become

$$B C = \frac{1}{S}\left(\sqrt{\frac{\sigma}{s}\,(a^2 s\,\sigma + 2 P S)} - a\,\sigma\right),$$

and $A D = \frac{s}{\sigma} B C.$

NOTE 2.—*Investigation of the preceding formulæ.*—Let E be the point where A a, B b, would meet, if prolonged (this point is not shown in the figure), then,

$$A E = \frac{a\,s}{S},\ B E = \frac{a\,\sigma}{S},\ \text{area of the triangle } A B E = \frac{a^2 s\,\sigma}{2 S},$$

and the area of the triangle C D E $= \frac{a^2 s \sigma}{2 S} - A$. Now because the triangles A B E, C D E, are similar,

area Δ A B E : area Δ C D E : : B E² : C E², that is,

$$\frac{a^2 s \sigma}{2 S} : \frac{a^2 s \sigma}{2 S} - P : : \frac{a^2 \sigma^2}{S^2} : C E^2, \text{ whence}$$

$$C E = \sqrt{\frac{2 \sigma}{S s}\left(\frac{a^2 s \sigma}{2 S} - P\right)}, \text{ and}$$

$$B C = B E - C E = \frac{a \sigma}{S} - \sqrt{\frac{2 \sigma}{S s}\left(\frac{a^2 s \sigma}{2 S} - P\right)}, \text{ or}$$

$$B C = \frac{1}{S}\left(a \sigma - \sqrt{\frac{\sigma}{s}(a^2 s \sigma - 2 P S)}\right)$$

$$\text{Also } B E : B C : : A E : A D = \frac{A E}{B E} B C = \frac{s}{\sigma} B C$$

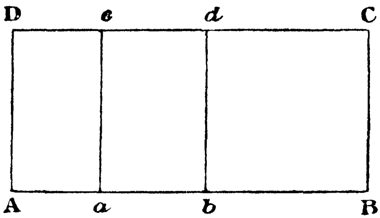
The other modification of the formulæ for B C, when the point E falls on the other side of A B, is derived by making S negative.

Case VIII.—*To divide a rectangular piece of ground of equal value throughout, either equally or unequally, among any given number of claimants, by fences parallel to one of its sides.*

A.—If the parts into which the rectangular space is to be divided be equal, it will be only necessary to divide two of the opposite sides of the rectangle into the given number of equal parts, and range the lines for the several fences to the consecutive points of division, and the land will then be divided as required.

B.—If the rectangular space is to be divided among several joint purchasers, who have paid unequal sums for the purchase thereof; then use the following:

RULE.—As the sum of all the sums paid, is to the length of the side of the rectangle from which the division lines abut, so is any one person's sum to the breadth on the side of the rectangle due to that person; then divide both this side and the one opposite to it, by laying off the resulting breadths, and range lines to the corresponding points, and the rectangle will be divided by parallel lines, as required.



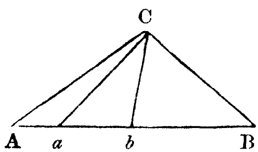
Example.—Divide the rectangle A B C D, the length of which is 1470 links and its breadth 684 links, among three joint-purchasers P, Q and R, who paid for the purchase thereof respectively £120, £150, £220.

£120
150
220
———

		links		£		links
490	:	1470	: :	120	:	360 = A a = P's breadth
——	:	——	: :	150	:	450 = a b = Q's ditto
——	:	——	: :	220	:	660 = b B = R's ditto

Whole breadth 1470 which proves the work.

Case IX.—*To divide a triangle of equal value throughout, either equally or unequally, among several claimants, who shall all have the use of the same watering-place, situated at one of the angles of the triangular field.*



Example.—It is required to divide the triangular field A B C among three persons, whose claims therein are as the numbers 2, 3 and 5, so that they may all have the use of a watering-place situate at the angle C; A B being = 1000, A C = 685, and C B 610 links.

The rule in this Case is the same as in the last.

$$\begin{aligned} \text{As } 2+3+5 = 10 &: 1000 :: 2 : 200 = Aa, \\ — &: —— :: 3 : 300 = a\,b, \\ — &: —— :: 5 : 500 = b\,B; \end{aligned}$$

which are the portions of the base A B, belonging to the respective claimants; therefore, if lines be drawn from a and b to C, the triangular field will be divided in the required proportion, each claimant having the use of the watering-place at C.

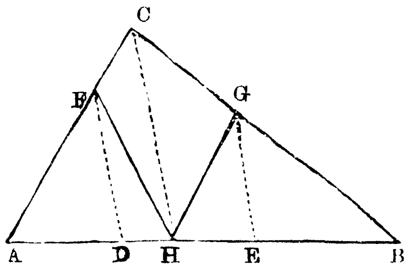
NOTE.—In dealing with this Case it is not necessary to use

the lengths of the side A C, C B, because all the three triangles A C *a*, *a* C *b*, *b* C B have a common perpendicular, and therefore their areas are as their bases.

Case X.—*To divide a triangular field of equal value throughout, either equally or unequally, among sundry claimants, by fences running from any given point in one of its sides.*

This is best explained by the following

Example.—Divide the triangular field A B C, the sides of which measure 30 chains = A B, 23 = B C, and 19 = A C, equally among three persons, by fences running from an occupation road that meets the side A B at H, which is 14 chains from A, that all the three persons may have the use of the road at H.



Divide A B into three equal parts in the points D, E; from H (the point where the road meets A B), draw H C; parallel to which draw D F, E G meeting A C, B C respectively, in F and G; and join H F and H G, in which directions fences being made will divide the triangle as required.

NOTE.—This method is founded on the areas of triangles between the same parallels being proportional to their bases; but it may be effected by actual measurement on the ground, by means of the following preliminary calculation:

Another Method.

As A H : A C : : A D = $\frac{1}{3}$ A B : A F,
that is, 1400 : 1900 : : 1000 : 1357$\frac{1}{7}$ links = A F;
and H B : B C : : E B = $\frac{1}{3}$ A B : B G,
that is, 1600 : 2300 : : 1000 : 1437$\frac{1}{2}$ links = B G,
whence the distances A F and B G may be measured from A and B; and from the points F and G the fences of division may then be drawn to H.

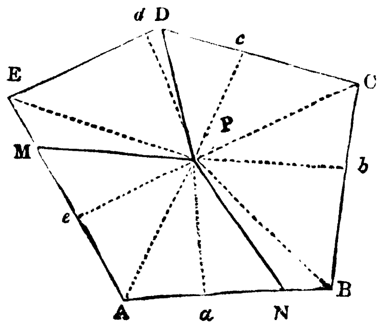
NOTE.—When the claims of the several persons are unequal, it will be readily seen that A B is then only required to be divided in the proportion of the several claims, as in the

preceding case, after which the method will be the same as that just given.

Case XI.—*To divide an irregular field with any number of sides among sundry claimants, so that they may all have the use of a pond, situated at a given point within the field.*

The following example explains the method of working:—

Example.—Three persons, Q, R, and S, bought a five-sided field A B C D E, having a pond therein at P, for which they paid respectively £100, £150, and £200; it is required to divide the field into parts, in proportion to each person's claim, and so that each may have the use of the pond P, the quality of the land being equal throughout. The lengths in links of the sides and the perpendiculars on each of them to the centre of the pond P are as below, thus constituting five triangles:—



A B = 864	P a = 560
B C = 827	P b = 608
C D = 806	P c = 480
D E = 682	P d = 544
E A = 990	P e = 540

By multiplying each side by half the perpendicular thereon, the sum of the five products will be the area of the field, thus,

$$
\begin{aligned}
864 \times 280 &= 241920 = \text{area of A P B},\\
827 \times 304 &= 251408 = \text{——— B P C},\\
806 \times 240 &= 193440 = \text{——— C P D},\\
682 \times 272 &= 185504 = \text{——— D P E},\\
990 \times 270 &= 267300 = \text{——— E P A},
\end{aligned}
$$

sum = 11·39572 acres, the area of the field.

The sums paid for the field by Q, R, and S, are as the numbers 2, 3, and 4, the sum of which is 9, therefore,

9	:	1139572	: :	2	:	253238	=	Q's share.
—	:	———	: :	3	:	379857	=	R.s do.
—	:	———	: :	4	:	506476	=	S's do.

Let D P be assumed to be the divisional fence between Q and S's shares; then the area of the triangle D P E = 1·85504 acres less than Q's share, therefore

$$\begin{array}{rl} 2{\cdot}53238 & = \text{Q's share.} \\ 1{\cdot}85504 & = \text{area of D P E,} \\ \hline {\cdot}67734 & = \text{difference.} \end{array}$$

This difference must be taken from the triangle E P A to complete Q's share; this is best done by dividing the said difference by half the perpendicular P e of the triangle E P A, and the quotient will be the distance E M, thus,

$\frac{1}{2}$ P e = 270) 67734 (250·87 links = E M.

The distance E M, being nearly 251 links, must now be measured from E on E A, which will give the point M, and, a straight fence being set out from M to P, will cut off Q's share.

The remainder of the triangle E P A, viz., 2·67300 − ·67734 = 1·99566 less than R's share, therefore,

$$\begin{array}{rl} 3{\cdot}79857 & = \text{R's share,} \\ 1{\cdot}99566 & = \text{area M P A,} \\ \hline 1{\cdot}80291 & = \text{difference.} \end{array}$$

This difference must be taken from the triangle A P B to complete R's share, as before, thus,

$\frac{1}{2}$ P a = 280) 1·80291 (643·9 links = A N.

This distance being measured from A towards B, will give the point N; and, the fence N P being now set out, will divide the field as required; the triangles N P B, B P C, C P D making up the exact quantity required for S's share, as may be readily shown by adding their three

areas together, which will prove the accuracy of the work, thus,

	864	= A B
	643·9	= A N
difference	220·1	= N B
	280	= $\frac{1}{2}$ P a
	61628·0	= area N P B
	251408	= —— B P C
	193440	= —— C P D
sum	506476	= S's share, which proves the work.

NOTE 1.—If some or all of the fences of the field A B C D E had been crooked, the operation of division would have been the same, excepting that the quantities of the offsets would have to be taken into the account, thus making a little additional calculation. It will at once be seen that this method of division may be extended to any number of claimants, whatever be the shape of the ground to be divided, the dotted lines from P to the angles of the field not requiring to be measured.

NOTE 2.—In this case the division is effected and approved without the aid of a plan, but it would, perhaps, be better for the satisfaction of the claimants, as well as for the surveyor himself, to plan the whole of the work; especially as an error in the work might thus be more readily detected.

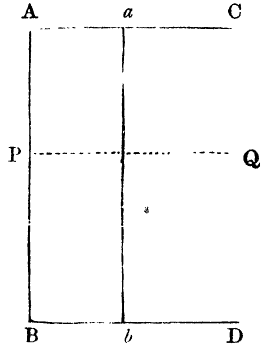
Case XII.—*To set out from a field of variable value, a quantity of land, that shall have a given value, by a straight fence in a given direction.*

NOTE.—This case presents a great variety of cases, the most simple of which is first produced.

A.—C A B D is a portion of a straight-sided field, right angled at A and B, and P Q a line, also at right angles to A B, dividing the field into portions of different values; it is required to lay off a quantity of land a A B b of given value, by a straight fence a b parallel to A B.

RULE.—Multiply A P, P B, respectively, by the value of the adjoining land, take the sum of the products, divide the

value of the whole portion to be laid off with five cyphers annexed by this sum, and the quotient will be the required breadth A a or B b.



Example.—Let A P = 500, P B = 700 links; the value of the land between A C and P Q £50, and that between P Q and B D £80 per acre; it is required to lay out land to the value of £300, by a straight fence $a\,b$ parallel to A B.

$$\text{Here } 500 \times 50 = 25000$$
$$700 \times 80 = 56000$$
$$81000 = \text{sum of products; and}$$

$$81{\cdot}000 \left\{ \begin{array}{l} 9)30000{\cdot}000 = \pounds 300 \text{ with five cyphers annexed,} \\ \quad 9)3333{\cdot}3' \\ \qquad 370{\cdot}37 = 370\tfrac{1}{3} \text{ links nearly} = \text{A } a = \text{B } b. \end{array} \right.$$

NOTE 1.—If the "quality" line P Q be not perpendicular to A B, it may be made so by "giving and taking;" especially as the required breadth A a may be nearly known by a rough calculation.

NOTE 2.—This case is of great importance, as land of variable quality is very frequently required to be laid out, in the enclosure of extensive commons, by straight fences in given directions for the purposes either of drainage or of irrigation. If any crooked portion of the land to be enclosed, lie to the left of A B, it must be measured, valued, and deducted from the whole value of the land to be laid out; then the Rule, here given, may be applied to the remaining value.

NOTE 3.—If the three parallel straight lines A a, B b, P Q, are not at the same time perpendicular to A B, the Rule just given will equally apply, excepting that the distance of $a\,b$ from A B must be laid off perpendicularly to A B.

NOTE 4.—This rule for laying out land of variable quality has not, it is believed, been previously given, the method of performing the operation being invariably by approximating to the true quantity by means of "guess lines."

To investigate the rule, put $AP = a$, $PB = b$, $Aa = Bb = x$, m = value of land above PQ, n = value of land below it, V = whole value to be laid out, and l = square links in one acre: then $\frac{amx}{l}$ = value of land adjoining AP, and $\frac{bnx}{l}$ = value adjoining PB,

$$\text{therefore } \frac{amx}{l} + \frac{bnx}{l} = V\text{; whence}$$

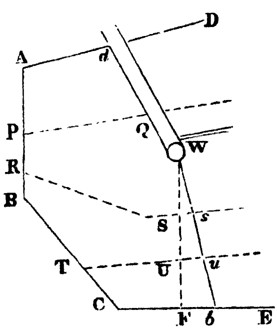
$$x = \frac{lV}{am + bn}$$

This rule may be obviously extended to any number of different qualities of land, bounded by parallel quality lines, by multiplying the breadth of each quality by its price, and taking the sum of all the products for a divisor.

B.—When the divisional line is required to pass through a given point W, where an occupation road and watering place is situated, which is to be used by the owners of three or more shares of a common of variable value.

The method of proceeding in this case will be best understood from the following

Example.—Let D A B C E be a portion of a common of variable value, W a watering place, and dW a road, forming part of the divisional line: it is required to set out a quantity of land of a given value V by a straight fence W b; m being = value of land between PQ and R S s, n = value between R S s and T u, o = value between T u and C E, and p = value between PQ and A d, the quantity of which is given by boundaries already determined.



First draw W F perpendicular

to C E, and find the value of the land to the left of d W F, let this value $= v$ which in this case is assumed to be less than the given value V′; therefore, the value of land required to the right of W F will be V $- v$, which put $=$ V′, and having adjusted the portions of the quality lines S s, U u perpendicular to W F, as per Note 1, **A.**, page 167, let W S $= a$, S U $= b$, U F $= c$, their sum, or the whole distance, W F $= s$, and F $b = x$, the symbols denoting the different values of the land being given in the example, and $l =$ square links in an acre: then

$$x = \frac{2\, l\, s\, V'}{a^2 m + (2a + b)\, b\, n + (2a + 2b + c)\, c\, o} = F\, b$$

which distance being measured, the straight fence W b may now be set out, and the other shares to the right of d W b may be next proceeded with, according to the method given in this or the following cases:

THE ABOVE EXAMPLE IN NUMBERS.

W S $= a = 460$, S U $= b = 400$, U F $= c = 420$ links; the values are $m =$ £30, $n =$ £40, and $o =$ £60 per acre, and the value V′ $=$ £80: required the distance F b or x by the preceding formula.

Here $s = a + b + c = 460 + 400 + 420 = 1280$ links, whence

$$= \frac{2 \times 1280 \times 100000 \times 80}{460^2 \times 30 + (920 + 400) \times 400 \times 40 + (920 + 800 + 420) \times 420 \times 60}$$

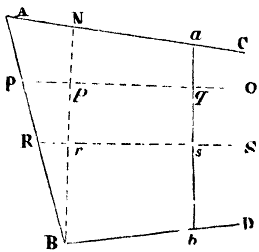
$$= \frac{20480000{\cdot}000}{81396{\cdot}000} = 251{\cdot}6 \text{ links} = F\, b,$$

which determines the position of the fence W b.

NOTE.—This method possesses the advantage of being practicable on the ground without the help of either a map or guess-line; however, in the case of the division of commons a map is always necessary for the satisfaction of the several parties interested therein. The investigation (see page 205) of the above formula is founded on similar triangles combined with the same principle as that in **A**, page 168. If there be less than three different qualities of land to be laid out, the symbols referring to the additional qualities, must be made to vanish in the formula; and, if there be more than three qualities to be laid out, the law for the extension of the formula is obvious.

C.—To set out from a field or common of any form, and of variable value, a quantity of land of given value by a straight fence in a given direction.

Example.—Let C A B D be a portion of a common, of variable value, from which it is required to set out a quantity of land of a given value V, by a straight fence $a\ b$ parallel to B N, the quality lines P Q, R S being so adjusted as to be perpendicular to B N at p and r respectively, as per Note 1, **A**, page 167, and the value of the land between A C, P Q being $= m$, between P Q, R S $= n$, and between R S, B D $= o$.



First find the value of the land in the triangle A N B, and if the fence A B had been crooked, the offsets would have to be included; the value of the land to the left of B N being supposed to be less than is required to make up the given value V. Let the value of the land in the triangle A N B $= v$, then the value of the land still remaining to be set out will be $V - v$, which put $= V'$, let $N\,p = a$, $p\,r = b$, $r\,B = c$, $p\,q = r\,s = x$, cot. of the angle C N B $= 2\alpha$, and cot. of the angle D B N $= 2\beta$, the symbols denoting the different values of the land being already given in the example, and $l =$ square links in an acre; then

$$x = \frac{2\,l\,V'}{am + bn + co + \sqrt{(am + bn + co)^2 - 4(\alpha m + \beta o)\,l\,V'}} = pq \text{ or } rs$$

which distance may be set out at any two points perpendicular to N B; and, if the land in question be a common, requiring the shares of several claimants to be set out, the quality lines may be again adjusted to the right of $a\ b$, and the next share may then be set out as before.

NOTE 1.—The investigation (see page 205) of the general formula just given, is the same in principle as those in the preceding cases, being only a little more complex, on account of the land to be set out not being rectangular, and therefore involving the solution of a quadratic equation.

NOTE 2.—When one or both of the angles C N B, D B N are obtuse, their $\frac{1}{2}$ cotangents α and β will be one or both negative respectively; and when these angles are right

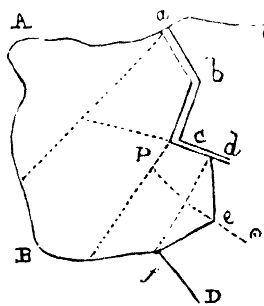
angles, their cotangents vanish; and the formula for the perpendicular breadth $p\,q$ or $r\,s$ becomes

$$x = \frac{l\,V'}{a\,m + b\,n + c\,o} = p\,q = r\,s$$

which is the same as the formula given in **A**, page 168, as it obviously ought to be.

D.—When the boundary of the land to be laid out is very irregular, also when part of the divisional fence is predetermined, either for the purpose of drainage or irrigation, and when the quality lines run irregularly.

Example.—Let C A B D be the boundary of a portion of common, from which it is required to lay out a portion of land of variable value, part of the divisional fence of which it is desirable to have in the direction of a watercourse $a\,b\,c\,d$, and the remainder of the divisional fence to run from d to f.



Join $d\,f$, and find the value of the land included thereby and the other boundaries in question, and, this value being found less than the required value, the remaining land still required may be determined by assuming two lines, so as to form a triangle on $d\,f$, as a base, till the correct position of the remaining part $d\,e\,f$ of the divisional fence shall be ascertained, it being advisable to let the angle e, in the required fence, fall on the quality line P Q, for the purpose of more readily calculating the areas of the two triangles formed thereby, and from thence finding their values.

Case XIII.—A.—*To divide a common of uniform value among any number of proprietors, in proportion to the values of their respective estates.*

In this case the quantities and values per acre of each proprietor's estate must be determined, by survey if neces-

sary, and the plan with the quantity of the common, must also be prepared; the value of the common being uniform, is not required.

RULE.—As the sum of the values of each person's estate : the whole quantity of common to be divided : : the value of each person's estate : to that person's share.

Example.—Divide a common of 100 acres among three persons, A, B, and C; A's estate is 120 acres at £4 per acre, B's is 180 acres at £5 per acre, and C's £260 acres at £2 per acre.

		£		£
120	×	4	=	480
180	×	5	=	900
260	×	2	=	520

——		acres				*a.*	*r.*	*p.*		
1900	: :	100	: :	480	:	25	1	2.1	=	A's share.
——	: :	——	: :	900	:	47	1	18·9	=	B's do.
——	: :	——	: :	520	:	27	1	18·9	=	C's do.
						99	3	39·9		proof.

The quantity of each person's share being now determined, the common may be divided by one or other of the methods already given.

B.—*To divide a common of variable value among any number of claimants, in proportion to the values of their respective estates.*

In this case, the plan of the common with the quantity of every different quality of land thereon, must be prepared, as well as the quantities and qualities of the several parts of each claimant's estate, in order that the values of both the common and each person's estate may be properly ascertained; the quality lines on the common being marked out by the valuer or valuers, previous to the survey being made.

When the survey of the common is completed and mapped, with the quality lines laid down thereon, the several qualities must be numbered 1, 2, 3, &c., with the values of the land corresponding to each number, as determined by the valuers. Most surveyors use the letters of the alphabet to denote the different values of the land: thus *a* may represent five shillings, *b* six shillings, &c., per acre per annum. By using the letters in this manner a multiplicity of figures, which may be confounded with other numbers, is avoided.

Before the lands of a common or waste can be divided and allotted, both public and occupation roads must be set out in the most convenient manner; they should be straight and, as far as practicable, at right angles to one another, as this arrangement not only facilitates the division of the land, but contributes greatly to the economy of cultivation with the plough. All old roads that may be deemed unnecessary may be stopped up and allotted to the different claimants, or diverted into more convenient directions, at the discretion of the Commissioners.

Portions of the common are now to be set apart for quarries, sand or gravel pits, if such exist in the common. The ground, thus set out, is considered as the common property of the several claimants, for the purpose of building, making roads, &c. Also, if there are any good springs or ponds on the common, they must be left unenclosed, in like manner, for common use; or the water must be conveyed from them by pipes or channels to more convenient situations, previous to the enclosure of the common.

The lord of the manor in some places claims $\frac{1}{12}$th of the common, in some $\frac{1}{16}$th, &c. His claim, whatever it may be, must be next set out, after its value has been determined from the whole value of the common. The lord of the manor will also be further entitled to his share of common, in proportion to his property, in the same manner as the other proprietors.

Lastly, when the roads, watering places, quarries, sand and gravel pits, and manorial rights have been set out, the remainder of the common must be divided equitably, as respects quantity, quality, and situation, among the proprietors of lands, tenements, houses, cottages, &c., situated in the parish or township where the enclosure is to be made.

RULE.—Having found the sum of the values of each proprietor's estate, and the whole value of the remaining part of the common to be divided, proceed to find the value of each proprietor's share as follows: As the sum of the value of each proprietor's estate, is to the whole value of the common remaining to be divided, so is the value of each proprietor's estate to his share of the value of the common.

It is unnecessary to give an example in this case, as

the laying out of the shares of the several claimants, after their respective values have been found by the above rule, is only a repetition of the methods already given, on a large scale; and after the work of laying out the several enclosures on the ground has been completed, the last enclosure or share of the common must be of the same value as that assigned by the rule, or so very near to it that the error is of no importance · otherwise a mistake has been made which must be immediately rectified.

CHAPTER XII.

PLOTTING PLANS AND SECTIONS.

Drawing Instruments.—The simple drawing instruments in common use are familiar to all. It is false economy to purchase cheap ones, as the best can be obtained at a trifling extra cost.

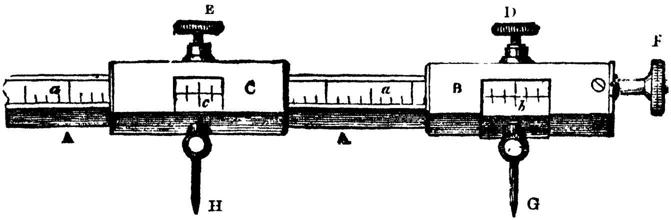
The author considers that the following only are required by the articled pupil at the commencement of his apprenticeship—viz., a drawing pen, with turn-up nib; dividers; 6-inch compasses, double jointed (one leg to carry needles, the other to carry ink and pencil points); set of spring bows with needle points; pricker; and 2-foot rule with divided scales. Such instruments may be carried in a leather case made for the purpose. There may also be provided with advantage a 12-in. rolling ebony parallel rule.

The remaining instruments constantly in use are generally provided by the surveyor, and consist of drawing board, which should be properly framed of well-seasoned pine; T-square and set-squares, 45° and 60°, of pear wood or mahogany, with ebony edges; well-finished 6-foot steel straight-edge, railway curves, French curves, beam compasses, and protractor.

Railway Curves are a series of thin rulers shaped to arcs of circles of various radii, usually from 1½ to 30 inches, and are used for projecting railway curves on plans, and to determine the radii of curves already projected. The radius of each curve is marked upon it in inches; and when, for example, a curve is applied to a plan, the scale of which is 5 chains to an inch, the marked radius must be multiplied by 5 to obtain the true radius of the curve: thus, if the radius marked on the curve be 16 inches, then $16 \times 5 =$ 80 chains = one mile, which is the radius of the curve.

French curves are similar, but with several curves on one ruler. They can be had in great variety of shapes and sizes.

Beam Compasses.—This instrument, shown in the sketch, consists of a beam A A, of any length required, generally made of well-seasoned mahogany. Upon its face is inlaid, throughout its whole length, a slip of holly or boxwood, *a a*, on which are engraved the divisions, in the best instruments to ·01 of an inch. The Ordnance pattern here illustrated is divided to a scale of chains, 80 of which



occupy a length of 6 inches, which, therefore, represent 1 mile, 6 inches to the mile being one of the scales to which that survey is plotted.

Two brass boxes, B and C, are adapted to the beam; of which the latter may be moved, by sliding to any part of its length, and its position fixed by tightening the clamp screw E. Connected with the brass boxes are the points of the instrument, G and H, the point G being fitted with needle point, and H with pen or pencil, which may be made to have any extent of opening by sliding the box C along the beam, the other box B being firmly fixed at one extremity of the beam.

The object to be attained by the instrument is the nice adjustment of the points G H to any definite distance apart. This is accomplished by two verniers, or reading places, *b c*, each fixed at the side of an opening in the brass boxes, and affording the means of minutely subdividing the principal divisions, *a a*, on the beam, which appear through these openings. D is a clamp screw for a similar purpose to the screw E, that is, to fix the box B, and prevent motion in the point it carries after adjustment. F is a slow motion screw, by which the point G may be moved any minute distance for perfecting the setting of the instrument.

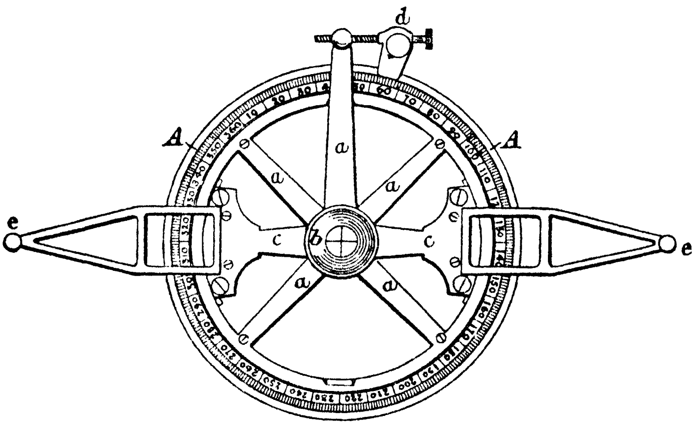
The method of setting the instrument may be understood from the above description of its parts, in conjunction with the following explanation of the method of examining and correcting the adjustment of the vernier *b*, which, like all other mechanical adjustments, will occasionally become deranged. This verification must be performed by means of a detached scale. Suppose, for example, that the beam compass is divided into feet, inches, and tenths, and subdivided by the vernier into hundredths, &c. First set the zero division of the vernier to the zero of the principal divisions on the beam, by means of the slow motion screw F. This must be done with great care. Then slide the box C, with its point H, till the zero on the vernier C exactly coincides with any principal division on the beam, as 12 or 6 inches. To enable us to do this with extreme accuracy, some superior kinds of beam compasses have the box C also furnished with a tangent or slow motion screw, by which the setting of the points of division may be performed with the utmost precision. Lastly, apply the points to a similar detached scale, and if the adjustment be perfect, the interval of the points G, H will measure on it the distance to which they were set on the beam. If they do not by ever so small a quantity, the adjustment should be corrected, by turning the screw F, till the points exactly measure that on the detached scale; then, by loosening the little screws which hold the vernier *b* in its place, the position of the vernier may be gradually changed, till its zero coincides with the zero on the beam; and the screws being now tightened, the adjustment will be complete.

The ordinary pattern of beam compass is supplied to fit any beam or lath, and has not the vernier adjustment. The best way of using the compass in this case is to lay down on a straight pencil line the length of the arc to be struck and set the compass to it.

The Circular Protractor.—There are various forms of the circular protractor, differing in detail. The sketch on the next page represents one of the best class.

This instrument is a complete circle A A, made from 6 inches to 14 inches in diameter, connected with its centre by four radii *a a a a*. The centre is left open and surrounded by a concentric ring or collar *b*, which carries two radial bars *c c*. To the opposite extremities of the bar

are fixed verniers which subdivide the primary divisions on the protractor to single minutes, and by estimation to 30 seconds. This vernier, as may be readily seen from the engraving, is carried round the protractor by turning the tangent screw *d*. Upon each radial bar *c c* is placed a branch *e e*, each point carrying at its extremity a fine steel pricker, whose points are kept above the surface of the paper by springs placed under their supports, which give way when the branches are pressed downwards, and allow the points to make the necessary puncture on the paper. The branches *e e* are attached to the bars *c c* with joints which admit of their being folded backwards



over the instrument, when not in use, and for packing in its case. The centre of the instrument is represented by the intersection of two lines, drawn at right angles to each other, on a piece of plate glass, which enables the person using it to place it so that the centre, or intersection of the cross lines, may coincide with any given point on the plan.

If the instrument is in correct order, a line connecting the fine pricking points with each other would pass through the centre of the instrument, as shown by the intersection of the cross lines on the glass; which, it may be observed, are drawn so nearly level with the under surface of the instrument as to do away with any serious amount of

parallax, when setting the instrument over a point, from which any regular lines are to be drawn. In using this protractor the vernier should first be set to zero, or the division marked 360, on the divided limb and then placed on the paper, so that the fine steel points may be on the given line, from whence the angular lines are to be drawn, and that the centre of the instrument may coincide with the given angular point in the same line. It is now ready to lay off the given angle, or any number of angles, that may be required from the given point, which is done by turning the tangent screw *d* till the opposite vernier reads the required angle. Then press the branches *e e* gently down, and they will cause their points to make the punctures in the paper, at opposite sides of the circle; which being afterwards connected, the line will pass through the given angular point, if the instrument was first correctly set. In this manner, at one setting of the instrument, any proposed number of angles may be laid off from the same point.

A cheap and useful protractor is that used in the Ordnance Survey department. It is 12 inches in diameter, of cardboard, with the centre cut out. The results are accurate on account of its large diameter.

Ink.—All plans (excepting finished architectural drawings, which may be in sepia,) should be drawn with the finest Chinese ink, prepared by rubbing down from the stick. Some skill is required in producing this of the proper degree of intensity of black. A very small quantity of water is put in the palette by dropping from a colour brush, and this small quantity made perfectly black; water may then be added from time to time until the sufficient quantity is prepared. The ink dries rapidly, and on no account should any fresh ink be mixed in the palette containing that first mixed, as lines from such ink would run if a colour wash were taken over them. A little powdered alum mixed with the ink will tend to prevent such running.

For making a tracing on which fine lines are not particularly requisite, the liquid Chinese ink supplied in small bottles by the makers may be used. The lines, however, have a tendency to be rubbed out if the tracing is subjected to wear.

Colours.—The author considers that colours supplied in the cake are more economical and cleaner in use than the pan colours. Some of the conventional colours used in plans are set out below—

Earthwork	Plan: burnt amber. ,, (turfed), light green.
Concrete	—Neutral tint mottled.
Brickwork	Elevation: lake, with yellow ochre. Plan: carmine or lake.
Stonework	Elevation: light sepia or Indian ink. Plan: dark Prussian blue.
Woodwork	—Burnt sienna.

Sections to be the same colour as the plan, but darker. A greasy surface such as parchment, before receiving ink or colour, may first be rubbed gently with a little prepared chalk, or clean blotting paper. Ox-gall, or even common soap, answers the same purpose, but ink mixed with ox-gall invariably runs if a colour wash is applied.

The colour brushes should be the best red sable.

Drawing and Tracing Cloths and Paper.—There is a great variety of these in the market, each more or less suitable for a specific purpose. For estate or engineering chief plans, the best Whatman's paper mounted on brown holland should be used; and for plans not receiving the same probable wear, paper varying in quality from Whatman's best to ordinary thin detail paper may be used, according to the importance of the drawing. Tracing cloths should be selected having regard to their transparency, closeness of the thread, and freedom from grease. The same remarks as to transparency and grease apply to the tracing papers, which also should be sufficiently tough to resist tearing.

Tracings are now generally made on the dull side of the cloth.

Pencils.—The best pencils for plotting the notes of a survey are A. W. Faber's first quality, Nos. 4 or 5. For all-round office work a No. 3 is perhaps more suitable.

Drawing to Scale.—Plans and sections are protracted on paper by means of scales. These scales are generally ratios of some given defined measurement to a certain unit of length on the plan or section—as, for example, a

scale of 40 feet to an inch; this indicating that a length of 40 feet on the ground is plotted on the paper in the length of one inch, and *vice versâ*, and all fractions of 40 feet on the ground are corresponding fractions of one inch on the paper.

In some cases the scale is expressed by the fraction which indicates the ratio of the length of the line on the paper to that of the corresponding line of the ground—as, for instance, the scale of the parish maps of the Ordnance Survey, which is $\frac{1}{2500}$. This ratio can of course be worked out to give the scale on the first method here referred to, and in the above example of the parish map is 25·344 inches to a mile, or 208·33 feet to one inch.

The scales are simply flat rulers of boxwood or ivory, with feather edges so as to lie flat on the paper, and bring the divisions close to it. They are about 12¼ ins. long, and give a clear divided space of 12 inches. This space is subdivided into primary divisions corresponding to the number of divisions required to the unit, as, for instance, in a scale of 4 chains to the inch, one inch would be divided into four parts, ¼ inch representing one chain. These primary divisions are again subdivided into ten, so that each subdivision on the scale in question will represent one-tenth of a chain or 10 links.

In the case of a scale of 40 feet to the inch, each of the primary divisions would represent 10 feet, and each of the subdivisions 1 foot. An offset scale is 2 inches long, divided exactly in the same way as the main plotting scale.

In plotting the survey the scale is laid at its zero point along the survey line on the paper, and the offset scale is placed in front of and moves along the main scale until the point is arrived at on the scale corresponding to that on the ground where the offset was taken. The length of the offset is then pricked off along the offset scale with a needle or the fine point of a hard pencil. It is, of course, impossible to subdivide the scales beyond certain limits, as the closeness of very fine divisions render readings difficult. Any subdivision required on the scales and not marked—as, for instance, 40 feet 6 inches—is estimated with the eye between the distances representing 40 and 41 feet respectively.

The scale to which a particular plan is drawn should be such as to show clearly the specific object for which it was

made, but no advantage arises from drawing to a larger scale than is necessary, for the larger the scale the larger the plan, and naturally the more labour. The Ordnance maps are drawn to scales described in Chapter XV., "Publications of the Ordnance Survey." The scale for a survey of an estate will vary from 2 to 10 chains to the inch, according to the size of the estate and the purpose for which the survey is made.

A plan of a building estate is conveniently drawn to 30, 33, or 44 feet to the inch. For engineering surveys a scale equal to that of the parish maps of the Ordnance Survey, 208·33 feet to the inch, is very useful, and for Parliamentary plans the scales must be those laid down in the standing orders.

Correction for Distances when Measured from Plan in Error by Wrong Scale.—It sometimes happens that a distance is scaled on a plan using by mistake a wrong scale, as, for instance, measuring with a 3-chain scale from a plan drawn to a scale of 2 chains to the inch. In such a case the error is adjusted by the ordinary rules of proportion. Assuming the distance to be measured with the 3 chain scale to be 241 feet, the correct distance (which would be measured by a 2-chain scale) is found thus:

$$3 : 2 :: 241 : \text{correct distance.}$$

The correct distance therefore equals $\frac{241 \times 2}{3} = 160$ ft. 8 in.

Instructions for Drawing Plans.—The paper on which the plan is to be drawn should be exposed for twenty-four hours in the office before use. The first step is to draw the scale on a convenient part of the paper, so that, in the event of any shrinkage taking place, the scale will be equally affected. The drawing table should have its edges rounded off to prevent creasing the paper.

In plotting a survey the main lines may conveniently be roughly plotted on tracing paper to ascertain the shape the plan will occupy on the paper. When this is arrived at, they may be carefully plotted in pencil on the paper. The filling-in lines should then be plotted, and subsequently the detail may be proceeded with, the offsets being plotted with the offset scale.

The detail when finished may be inked in, commencing at the left-hand top corner of the plan. When this is done the survey lines may be rubbed out, the commencement and end of them at their junction with any other lines being indicated by a faint red tick, so that they can be ruled in again should occasion require without re-plotting the whole work. The direction of the north should be marked on the plan. The lines for inking in should be very fine and all of one thickness. When the required thickness has been obtained the nibs of the pen should not be altered; they can be cleaned by passing a piece of paper between them.

The best light for this work is obtained in a room lighted from the top.

In colouring, the brush should be held straight, and the colour brought in an even wash towards the draughtsman, and great care taken not to exceed the limit of the area to be coloured.

The printing on plans is of very great importance to give a good appearance. That known as block printing and italic writing are the most useful, and an articled pupil will find great advantage in studying and copying the italic writing on the Ordnance maps.

Plotting a Survey made with the Magnetic Needle.—In a previous chapter on Surveying with the Theodolite, an example of a survey with the magnetic needle was given, and in the present chapter the protractor is described. The reader may now refer to the figure on page 96. To plot the bearings, place the protractor at *a*, so that *a b* shall have its due bearings with regard to the magnetic meridian N S, the north and south being represented on the protractor by 180°; prick off from *a*, the bearings of *a f*, *a b*, *b c*, *c d*, *d e*, *e f*; from *a* draw *a f* and *a b*, making the latter the length of the measured chain line, and from *b c d* and *e* draw with the parallel ruler *b c*, *c d*, *d e*, and *e f* parallel to their bearings pricked off from *a*, and make them of lengths corresponding to the measured chain distances.*

Plotting Sections.—In most cases, as already explained, surveying operations precede those of levelling. The horizontal distances can conveniently be plotted on the

* Heather's "Mathematical Instruments."

same scale as that of the plan, but this, though desirable, is not absolutely necessary. When the heights or vertical distances are plotted to the same scale as those horizontal, the section is known as a "natural" section. More often this cannot be obtained, as when the horizontal scale is small—for example, 3 chains to the inch: the measurement of vertical distances to such a scale would be uncertain and lead to serious error. In such cases the vertical distances are plotted to a different scale, as, for instance, 10 or 20 feet to an inch, the height being thus exaggerated to render measurement more convenient.

In plotting the section the datum line should first be laid down in ink, in a very fine line, and the horizontal measurements taken pricked off along it. Lines vertical to the datum line are then ruled in pencil from these points with a set-square, and the heights above the datum pricked off on them. These prick holes are subsequently joined, and represent the surface of the ground. For example, suppose the level of a point on the surface to be 25 feet about Ordnance datum, and for convenience a datum line 10 feet above Ordnance be fixed for the datum of the section. The height would be laid off with the scale so as to measure 15 feet above the datum line.

The section is then finished as shown in the drawings of the section in the chapters on Levelling and Railway Surveying.

Cross-sections can be either plotted to the same scales as the main sections, or, as for Parliamentary purposes, to scales larger than the main scale.

Gradients.—A gradient is the degree of slope or rate of inclination of any surface, as, for example, a road, railway, or sewer. On page 136, the gradient of the proposed new railway is shown to be 1 in 100, indicating that the railway in 100 feet—measured horizontally—rises or falls 1 foot, measured perpendicularly.

The method of laying out gradients is by applying one end of an extended silken thread to the section at its commencement, the other end being so applied that the thread may cut the profile of the earth's surface, so as to leave equal portions of space both above and below the thread, as nearly as can be estimated by the eye, so that the cuttings from the parts above the thread may furnish sufficient

materials to fill up the spaces or parts below the thread to form the embankments. A straight edge of glass may also be conveniently used for this purpose.

If the position of the first gradient, though in itself favourable, should cause the next gradient to be less favourable, with regard to the extent of cuttings and embankments, the position of the first gradient must be altered to suit the next following gradient, till it be found that the compound result of the cuttings and embankments on the two gradients, or on the successive gradients, as now altered, shall have less of cuttings and embankments than in the preceding case. In this way it is advisable to change the positions of the gradients till the *minimum* of cuttings and embankments seems evidently to be attained, due regard being had to the limit of safety in the ascent or descent of the gradients, also keeping in view, at the same time, the proper height for bridges to cross rivers, &c., the difficulty of making the excavations being supposed equal.

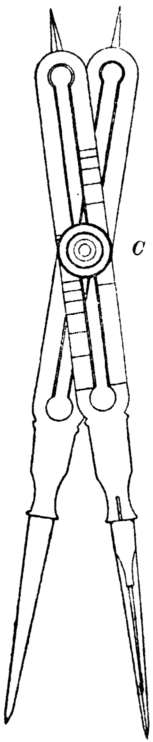
Rule for finding the Rate of Inclination of a Gradient.—Divide the horizontal length of the gradient in feet, by the difference of the heights of the gradient at its extremities, above the datum line, and the quotient is the horizontal measurement to a rise or fall of one foot. As above stated this is the rate of inclination of the gradient.

Example.—Assume the gradient at its commencement, to be 261·35 feet, and at its termination to be 269·20 feet above the datum line; thus giving a rise of 269·20 − 261·35 = 7·85 feet, and assume the horizontal length of the gradient is to be 31·50 chains = 2079 feet. Then 2079 ÷ 7·85 = 264 (fractions omitted), or a rise of 1 in 264, or of 20 feet per mile.

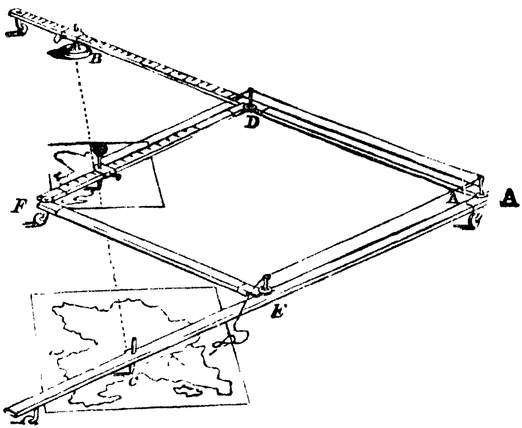
Enlarging and Reducing Drawings. — Various methods are in use for enlarging and reducing drawings, all approximating nearly to accuracy of results. If the utmost accuracy is indispensable, and the survey notes are at hand, the best method is to re-plot the work to the scale required. The methods referred to are :—

(1) By proportional compasses.
(2) By the pentagraph.
(3) By the eidograph.
(4) Bv the method of squares.

(1) *By proportional compasses.*—This instrument is supplied with a good set of drawing instruments, and when the drawing to be dealt with is small may be used. The illustration will explain the instrument, the bar C sliding between the legs of the compasses and being fixed in any position by means of the milled-headed screw.



(2) *By the pentagraph.*—The pentagraph consists of four brass bars, A B, A C, D F, and F E; the two longer bars, A B, A C, are connected by a movable joint at A; the two shorter bars are connected in like manner with each other at F, and with the longer bars at D and E, and, being equal in length



to the portions A D, A E of the longer bars, form with them an accurate parallelogram A D F E, in every position of the instrument. Several ivory castors support the machine parallel to the paper, and allow it to move freely over it in all directions. The arms A B and D F are graduated and marked $\frac{1}{2}$, $\frac{1}{3}$, &c., and have each a sliding index which can be fixed to any of the divisions by a milled-headed clamping screw, seen in the engraving. The sliding indices have each of them a tube, adapted either to slide on a pin, rising from a heavy circular weight called the fulcrum, or to

receive a sliding holder with a pencil or pen, or a blunt tracing point, as may be required.

When the instrument is correctly set, the tracing point, pencil, and fulcrum will be in one straight line, as shown by the dotted line in the figure. The motions of the tracing point and pencil are then each compounded of two circular motions, one about the fulcrum, and the other about the joints at the ends of the bars, upon which they are respectively placed. The radii of these motions form sides about equal angles of two similar triangles, of which the dotted straight line B C, passing through the tracing point, pencil, and fulcrum, forms one side; hence the distances passed over by the tracing point and pencil, in consequence of either of these motions, have the same ratio, and, therefore, the distances passed over in consequence of the combination of the two motions, have also the same ratio, which is that indicated by the setting of the instrument.

The engraving represents the pentagraph in the act of reducing a map to the scale of half the original. For this purpose the sliding indices are first clamped at the divisions on the arms, marked $\frac{1}{2}$; the tracing point is then fixed in the socket at C, over the original map; the pencil is next placed in the tube of the sliding index, on the bar D F, over the paper to receive the copy; and the fulcrum to that at B, on the bar A B. The machine being now ready for use, if the tracing point C be passed delicately and steadily over every line of the map, a copy, but of one half of the scale of the original, will be marked by the pencil on the paper beneath it. The fine thread represented as passing from the pencil quite round the instrument to the tracing point C, enables the draughtsman at the tracing point to raise the pencil from the paper while he passes the tracer from one part of the original to another, and thus to prevent false lines being made on the copy. The pencil holder is surmounted by a cup, into which sand or shot may be put, to press the pencil more heavily on the paper, when found necessary.

If the object were to enlarge the map to double its scale, then the tracer must be placed on the arm D F, and the pencil at C; and, if a copy were required of the same scale as the original, then, the sliding indices still remaining at the same divisions on D F and A B, the fulcrum must take the middle station, and the pencil and tracing point those on the exterior bars A B, A C of the instrument.

Though the pentagraph affords the most rapid means of reducing a map or drawing, we cannot recommend its use for enlarging a copy, or even for copying on the same scale, especially if the original drawing is a complicated one.

(3) *By the eidograph.*—This instrument is more reliable, owing to its construction, than the pentagraph. It has the disadvantage of being expensive, a 30-inch eidograph costing £14. The instrument has only one point of support on the paper, the joints being fulcrums fitting in accurately ground bearings. It can be set to enlarge or reduce a drawing in any proportion, thus differing from the pentagraph, which can only work according to the divisions engraved on it.

(4) *By the method of squares.*—This method, although in reducing sufficient accuracy is secured, is at best laborious. The method may be best shown by an example. Let Fig. 1, of the two annexed sketches, represent a plan of an estate which it is required to copy upon a reduced scale of one-half. The copy will therefore be half the length and half the breadth, and consequently will occupy but one-fourth the space of the original.

Fig. 1.

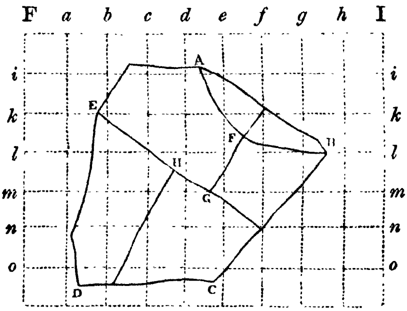
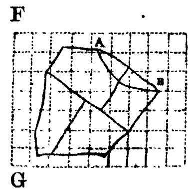


Fig. 2.



Draw the lines F I, F G at right angles to each other; from the point F towards I and G, set off any number of equal parts, as F *a*, *a b*, *b c*, &c., on the line F I, and F *i*, *i k*, *k l*, &c., on the line F G: from the points in the line F I, draw lines parallel to the other line F G, as *a a*, *b b*, *c c*, &c., and from the points on F G, draw lines parallel to F I, as *i i*, *k k*, *l l*, &c., which being sufficiently extended towards I and G, the whole of the original drawing will be covered

with small and equal squares. Next draw upon the paper intended for the copy, a similar set of squares, but having each side only one-half the length of the former as is represented in Fig. 2.

It will now be evident that if the lines A B, B C, C D, &c., Fig. 1, be drawn in the corresponding squares in Fig. 2, a correct copy of the original will be produced, and of half the original scale.

Commencing then at A, observe where in the original the angle A falls, towards the bottom of the square marked *d e*. In the corresponding square, therefore, of the copy, and in the same proportion towards the left-hand side of it, place the same point in the copy: from thence, tracing where the curved line A F crosses the bottom line of that square, which crossing is about two-fifths of the widths of the square from the left-hand corner towards the right, and cross it similarly in the copy. Again, as it crosses the right-hand bottom corner in the second square below *d e*, describe it so in the copy; find the position of the points similarly where it crosses the lines *f f* and *g g*, above the line *l l*, by comparing the distances of such crossings from the nearest corner of a square in the original, and similarly marking the required crossings on the corresponding lines on the copy. Lastly, determine the place of the point B, in the third square below *g h* on the top line; and a line drawn from A in the copy, through these several points to B, will be a correct reduced copy of the original line.

Proceed in like manner with every other line on the plan and its various details, and (assuming, of course, that the work is done with sufficient care and accuracy) the plot or drawing will be laid down to a small scale, yet bearing all the proportions in itself exactly as the original.

The process of enlarging drawings, by means of squares, is a similar operation to the above, excepting that the points are to be determined on the smaller squares of the original, and transferred to the larger squares of the copy. The process of enlarging, under any circumstances, does not, however, admit of the same accuracy as reducing.

Copying Drawings. — Several methods of copying plans and drawings are in use, as follows :—

Copying by tracing.—This is the most common method, a sheet of transparent tracing paper or cloth being placed

over the drawing, the lines of which, visible through the transparent medium, are duly marked on it.

By transfer paper.—A sheet of tracing paper, having its under side rubbed over with powdered black lead, is laid upon the paper intended to receive the copy; the original is then placed over both, and a tracing point is carefully passed over the lines of the drawing with a pressure proportionate to the thickness of the paper; and the paper beneath will receive corresponding marks, forming an exact copy.

Copying by reflected light through a copying glass.—In this method the drawing is placed on a large sheet of plate-glass, called a copying glass, and the paper to receive the copy placed over the drawing. The glass is then fixed in such a position as to have a strong light to fall upon it from behind, and to shine through it, and both the original drawing and the paper to receive the copy. By this means the lines of the original drawing become visible through the paper to receive the copy, which can be made with ease, without any risk of soiling or injuring the original.

Copying by pricking off.—This process is very commonly used. The plan to be copied is placed over a sheet of paper, and all points, forming the divergence or commencement of lines, are pricked through on to the paper beneath by means of a very fine needle, known as a pricker, supplied with every case of instruments.

Copying by the pentagraph, eidograph, and method of squares.—These methods for enlarging and reducing have been already described, and it will be readily seen how they can be adapted for producing a copy the same size as the original.

Photography, as well as *heliography* or *sun printing*, is now commonly resorted to where more than one copy is required, the advantage being that any number of copies can be prepared from one tracing.

Heliography is practised in three ways—namely, by the ferro-gallic process, in which the lines appear black on a white ground; by the ferro-prussiate process, in which the lines appear white on a blue ground; or by the ferri-cyanide process, in which the lines appear blue on a white ground.*

* Appliances and instructions for these processes may be obtained from Mr. W. F. Stanley, Great Turnstile, London, W.C.

CHAPTER XIII.

CALCULATION OF AREAS AND CUBICAL CONTENTS.

THE reader is referred to the chapters on Practical Geometry and Mensuration at the beginning of this work, where the rules, formulæ, and tables of measure necessary for ordinary calculation are set out. The object of the present chapter is to present the methods commonly in use among surveyors of applying such rules and formulæ in practice.

AREAS.

To reduce square links to acres.—RULE. Cut off 5 figures to the right; the remainder will be acres. Multiply the 5 figures by 4, and again cut off 5 figures; the remainder will be roods. Multiply the 5 figures by 40, and again cut off 5 figures; the remainder will be perches.

Methods of Determining Areas.

The following are the methods in general use:—

(1) By calculation, in certain cases applicable, without protracting the measurements on paper.

(2) By protracting the measurements, and, in the event of the enclosure having irregular boundaries, finding the area of the figure enclosed within the survey lines, and adding to it (or subtracting from it in case the survey line is outside the enclosure) the small triangles or trapeziums formed by the offsets and the space between the boundary of the enclosure and the survey line.

(3) By protracting the measurements, and reducing the irregular boundaries to straight boundaries by means of what are termed "give and take" lines, thus forming a

regular figure or an irregular polygon with a smaller number of sides than in the second case.

(4) By the computing scale.

(5) By the planimeter.

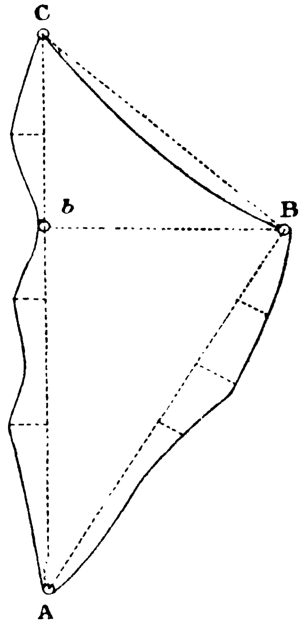
In all cases the position of the ditch with reference to the fence must be taken into account in calculating the area, as the line on the paper thus ———— represents the centre of the quicks of the fence, as already explained. (This matter is dealt with in that part of the chapter on Chain Surveying relating to boundaries, page 40.)

(1) *Areas by calculation only, without protracting on paper*:—In the case of a triangle, square, rectangle, parallelogram, circle, and its parts or ellipse, the area can be computed from the measurements, without plotting them on paper, by the ordinary rules of mensuration. Similarly, the area of a piece of land measured by equidistant offsets or ordinates, according to Simpson's rule (see page 19), can be ascertained. The area of a triangle can be computed from its three sides, as shown on page 16, or by trigonometry, as on page 81.

(2) *Areas from protracted measurements, without give-and-take lines*.—The ordinary method in this case, as also in case No. 3, is to make a tracing in ink of the enclosure, and to mark on it the allowance from the centre of the fence for the ditch. This, then, represents the actual extent of land in question, and the figure of a trapezium is split up into triangles—or, in the case of an irregular polygon, into triangles and trapeziums as most convenient—the measurements for the necessary calculation being taken from the plan with a scale. Such divisions of the enclosure are best marked with thin red lines, each subdivision (triangle or trapezium, as the case may be) being marked with a distinctive letter. An example of this method of computing is given below. It cannot compare with that in which give-and-take lines are used, as the areas of numerous small triangles and trapeziums have to be ascertained which in the other method are avoided.

Example.—Find the contents of an enclosure from the following field-notes :—

	to ⊙ A	
	2504	0
	2000	74
	1650	137
	1430	90
	1220	144
	850	30
	425	110
	000	0
	L. ⊙ C	
	to ⊙ C	
0	1346	
80	1072	
128	708	
98	458	
0	000	
	L. ⊙ B	
	to ⊙ B	1056 to ⊙ *b*
	1946	0
	1490	96
	1200	152
	1000	112
	600	
	520	50
	000	
From	⊙ A	go N. E.



Having drawn the figure, the proof line B *b* will be found to be 1056 links, as in the field-note.

Double areas.

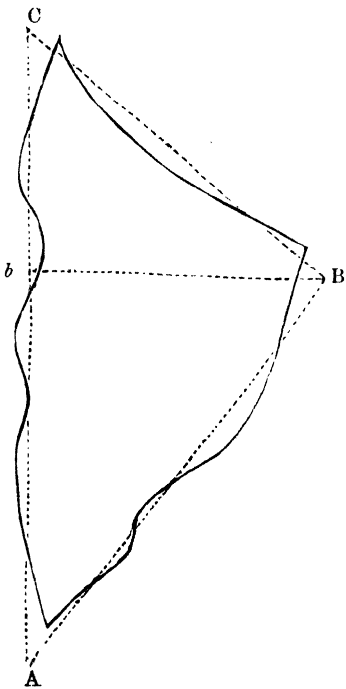
2644224	triangle A B C
653112	offsets on A B and A C
3297336	sum
199016	deduct areas on B C
2)3098320	difference

15·49160 = 15*a.* 1*r.* 38½*p.* nearly, the area required.*

(3) *Computation of the area by reducing the crooked sides to straight lines, by give-and-take lines.*—By this method, straight

* The observations in the chapter on Chain Surveying page 37) as to long offsets apply equally in this case.

lines are drawn on the plotted figure, so as to include as much space in the area which is to be measured as they exclude, as nearly as can be judged by the eye; the area to be measured is thus reduced to a figure bounded by straight lines only. The method of drawing these lines is usually by a straight edge of glass or ruler of transparent horn, or by a silken thread stretched, the ruler or thread being moved over the crooked fence, till it appear to the eye to enclose as much of the adjoining ground as is left out; a line is then drawn in this position, and so on for other crooked fences. Thus the trouble of calculating numerous offsets is completely avoided, and with proper care equal accuracy is obtained.



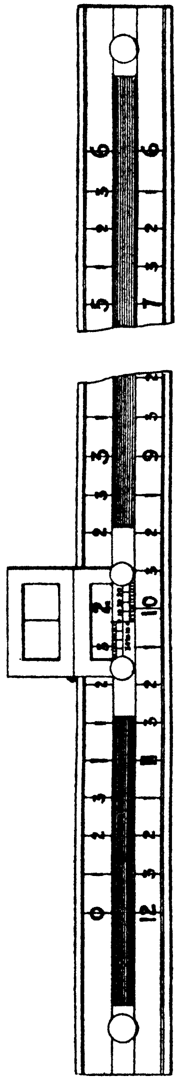
We shall adopt the last example for this method of taking out the area, that it may be seen how near the two methods agree.

A tracing of the figure being made, with the allowance from the centre of the fence marked on, the three lines A B, B C, C A must now be drawn, in such a manner, that the parts excluded by them may be equal to the parts included, as nearly as can be judged by the eye. The base A C will be found to be 2584 links, and the perpendicular B b = 1200 links. Whence

$$\frac{2584 \times 1200}{2} = 15{\cdot}50400 = 15a.\ 2r.\ 1p.$$

If the area found by the first method be taken from the

area just found, it will be seen that they differ by little more than one pole out of 15½ acres, or little more than 1 in 4000: thus



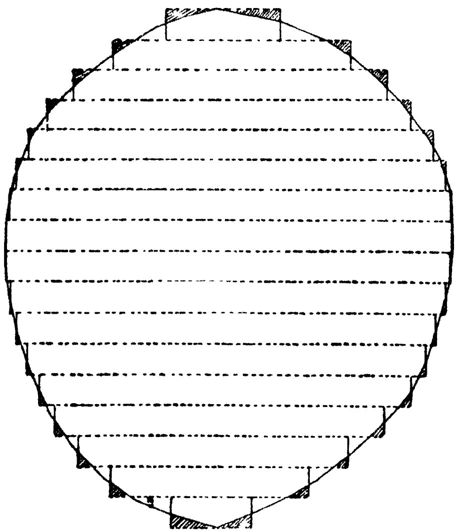
15·50400
15·49160
————
1240 square links, or a little less than two poles.

(4) *Areas by the computing scale.*—The computing scale is here shown. Areas can only be worked out from plans drawn to the same scale marked on the computing scale. A common scale is 3 chains to the inch. The computing scale can, however, be had divided as required.

It consists of a boxwood rule, in the case now described, of 20 inches long, and adapted to 3 chains to the inch; consequently the length of 20 inches will represent 60 chains, or 6 acres for a strip 1 chain wide. The length of the scale is, therefore, divided into 6 parts, each subdivided into 4 to represent roods as shown in the figure. The upper edge of the scale reads from left to right, viz., from 1 to 6 acres, and the lower, subdivided as the other from right to left, viz., from 7 to 12 acres. A metal frame arranged to lie flat on the paper slides in an undercut groove of the main scale. In the centre of the frame is an index of strained wire in line with the zero on a scale of perches on the frame, which are arranged to read from 0 to 40 from left to right on each side of the index, to correspond respectively with the main scale when read either on the lower or the upper edge. The space occupied on the frame of 40 perches is equal to that of one subdivision or 1 rood on the main rule.

The scale is used as follows:—A piece of tracing paper

is fastened over the plan, and on this tracing paper are drawn parallel lines 1 chain apart, equal in this case to $\frac{1}{8}$ inch. The computing scale, with its index at zero on the main scale, is placed on the first of the parallel lines on the paper enclosing any part of the estate. The scale must be placed in such a position that the index forms a vertical give-and-take line at the commencement of laying it on each parallel space. The frame is then moved along the scale until it arrives at the edge of the estate comprised within the first parallel space where a similar give-and-take



line is made. Without moving the index the scale is then taken up and placed over the second parallel space, and the operations described above are repeated until the index reaches the end of the rule, or 6 acres. The scale is then turned round so as to read with the lower edge, care being taken to place the index at the same spot on the paper where it registered the 6 acres. When the whole estate has been traversed the number of perches shown by the reading of the index, added to the number of acres and roods traversed, will give the area. The illustration shows the method of adjudging the give-and-take line.

(5) *Areas by the planimeter.*—This is an extremely delicate instrument, so liable to injury in ordinary use as to cause anxiety as to the accuracy of measurements made with it. It consists, essentially, of two arms jointed together so as to move with perfect freedom in one plane, and a wheel which is attached to one of the arms, and which, turning on this arm as an axis, records by its revolutions the area of a figure traced out by a point on the arm to which it is attached, while a point on the other arm is made a fixed centre about which the instrument revolves.*

Correction for Areas Measured for Plan, in error, by wrong scale.

Occasionally it may occur that an area is measured from a plan by mistake with a scale other than that to which the plan is actually drawn. To take an example, assume that a plan is drawn to a scale of 2 chains to the inch, and the dimensions from which the area is ascertained are accidentally scaled from the plan with a scale of 3 chains to an inch. The area (represented by 10 square inches on the plan) is apparently (though incorrectly) 9 acres, or 90 square chains, measuring with the 3-chains scale. The actual area is of course less, since an equal number of square inches on the plans drawn to 3 and 2 chains to the inch respectively, represent a less area on the ground in the case of the 2-chains scale than in that of the 3-chains scale.

The actual area is thus found by the ordinary rules of proportion, remembering that the units dealt with are in square measure :—

$$3^2 : 2^2 :: 9 \text{ acres} : \text{correct area}.$$

$$\text{Correct area} = \frac{9 \text{ acres} \times 2^2}{3^2} = 4 \text{ acres}.$$

CUBICAL CONTENTS.

The content of a solid rectangular body, such as a square block of stone, is easily computed. Before proceeding to calculations for estimating the contents of cuttings which are largely worked out by tables, a few simple examples of estimating cubical contents of common objects required in ordinary agricultural practice may be given.

* Heather's "Mathematical Instruments."

Excavations of Pits with Sloping Sides.

In the case of excavating for a pit, such as a marl pit, the sides are sloped back to prevent earth falling in, and then two cases are presented: (1) Where the bottom of the pit is taken out so as to be parallel with the surface. (2) Where the excavation is irregular.

Case I.—The excavation has the top and bottom forming two parallel plane rectilineal figures with its sloping sides trapezoids. It is, therefore, a prismoid, and the prismoidal formula, which is here repeated from the chapter on Mensuration for convenience, will apply.

RULE.—Add together the areas of the two ends, and four times the area of a section parallel to the two ends and midway between them, multiply the sum by the height, and $\frac{1}{6}$ of the product will be the volume.

Example.—Required the contents of a pit, the top of which is a rectangle 12 feet × 10 feet, the bottom section 8 feet × 6 feet, and the depth 4 feet.

Top section	$12 \times 10 = 120$	square feet.
Bottom section	$8 \times 6 = 48$	,,
Middle section	$10 \times 8 = 80$	,,

$$\text{Content} = \frac{[120 + 48 + (4 \times 80)] \times 4}{6} = 325\tfrac{1}{3} \text{ cubic feet.}$$

In this case the sides are regularly sloped, so that the middle section can be obtained by calculation. If the slope of the sides is not regular, the dimensions of the middle section must be determined by actual measurement.

The mean depth must be obtained by taking several equidistant depths, and ascertaining the average.

If the top and bottom of the pit are not rectangular, their areas, as also the area of the middle section, would have to be ascertained by the rules of mensuration relating to the particular figure presented by them.

Case II.—The rule for the contents of an irregular solid, quoted in the chapter on Mensuration, will apply, viz. :—Divide the figure into any even number of parallel and equidistant sections, and find the area of each section. Add

together the first area, the last area, twice the sum of all the other odd areas, and four times the sum of all the other even areas, multiply the sum by one-third of the length between each section.

Example.—Five equidistant sections of a solid are taken, the common distance being 10 feet, the areas of these sections in square feet are 37, 52, 69, 87, and 107 respectively for the volume of the solid.

$$\text{Volume} = [(37 + 107) + 2\,(69) + 4\,(52 + 87)] \times 3\tfrac{1}{3} \text{ cub. ft.}$$
$$= 2793 \text{ cub. ft.}$$
$$= 103{\cdot}4 \text{ cub. yds.}$$

Contents of Material Heaped on Surface.

The reader will be able to ascertain the contents of clay, or marl, or other materials heaped on the surface, by applying the ordinary rules of mensuration.

It will often be found that dimensions may be taken so as to get an average horizontal section and average depth by "giving and taking." In such cases the more laborious calculations are dispensed with.

Contents of Railway Cuttings, &c.

In making the estimates for a projected railway, the contents of the several cuttings, embankments, &c., are in most cases found by tables calculated for the purpose, the surface of the ground being considered as on the same level as the centre of the line. But when the projectors of the line have been empowered to construct it, cross-sections of the cuttings are carefully taken at every variation of the surface of the ground, especially if the surface be side-lying, or inclined laterally with respect to the direction of the line. The distance of these cross-sections may vary from 10 or 12 chains to less than 1 chain, according to the regularity or irregularity of the slopes of the surface. These cross-sections must next be plotted on a large scale, and their areas found by the ordinary rules of mensuration, preparatory to finding the contents of the cuttings by tables;

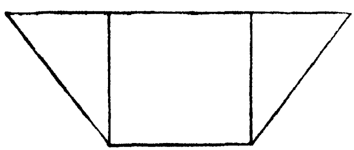
but if the surface lines of any of the cross-sections be level, or so nearly so as to be readily reduceable to a level surface line, their areas need not be found, their depth only being required for finding the contents by the tables.

For this purpose various tables have been published, such as Buck's "Handy General Earthwork Tables" (London: Crosby Lockwood & Son), and the tables given in Molesworth's "Pocket-Book of Engineering Formulæ" (London: E. & F. N. Spon).

Two sets of tables are mainly used, (1) in which the height or depth of the embankment or cutting, respectively, is given in feet, the length in Gunter's chains of 66 feet, and the contents in cube yards; (2) in which all the measurements are in feet, the length being in chains of 100 feet. It will be seen that this table is applicable to any measure when the length, height (or depth), and contents are of the same unit.

In addition to these tables, supplementary tables are given in some works, showing the contents on sidelong ground where a horizontal equating line is not drawn to give the height, but the areas are taken out from cross-sections.

The utmost accuracy must be observed in the measurements of height (or depth), and the calculation of the content therefrom, as an approximate height, varying say 6 inches from the true height, produces a very serious error if the length in question is considerable. The contents are, therefore, better calculated with measurements of height of every 3 inches. The contents are given in two columns in the tables—(1) for the central part, which is the frustrum of a wedge 1 chain long, and (2) for both slopes, which are frustrums of pyramids, as in the illustration.



For further elucidation of this subject, the student may be referred to Buck's "Tables."

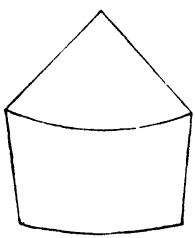
Contents of Stacks.

The reader is referred to the Tables of Measure, page 14, for hay and straw measure. The weight of hay varies as its density, which can be estimated correctly only after some experience. Stacks vary in weight from 8 stones to 16 stones per cubic yard. The weight of a stack can be estimated sufficiently accurately by cutting out a portion extending to the centre of the stack from the top to the bottom, weighing it and measuring the space it formerly occupied, and applying these data to the total number of yards contained.

Stacks are generally made in one or other of the several ways set out below.

Case I.—*To measure a stack straight from bottom to eaves and with circular base*

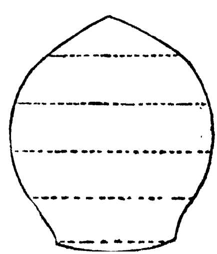
RULE.—Square the average circumference, multiply by ·08, and multiply the product of these dimensions by the perpendicular height to the eaves. This gives the content to the eaves.



For the conical top, square the circumference at the eaves, multiply by ·08, and by one-third of the perpendicular height from the top to the eaves.

Case II.—*To measure a stack bulged from top to bottom and with circular base.*

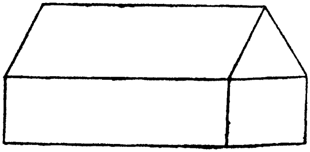
In this case Simpson's rule (see page 19) must be used, by finding the area of an odd number of circular sections (circumference^2 $\times$ ·08), taken at equal perpendicular distances from the bottom.



Multiply the area of the base of the top part remaining, after the last section is taken, by one-third the perpendicular height from this point to the top of the stack.

Case III.—*To measure a stack with rectangular base and perpendicular ends.*

Multiply the length by the average width (between bottom and eaves), and the product by the height from the ground to the eaves. For the top multiply the area at the eaves by half the height from this point to the top of the stack.

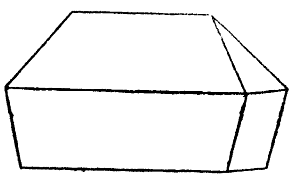


Case IV.—*To measure a stack with rectangular base, prismoidal body, and perpendicular ends.*

For the content of the body apply the prismoidal formula, see page 19. This is here reproduced for convenience. Add the area at the bottom to the area at the eaves. Add four times the area of a section parallel and midway between them, multiply the sum by the height to the eaves and divide by six. This gives the cubic content of the body. For the cubic content of the top, multiply the area at the eaves by one-half of the perpendicular height, from the eaves to the top of the stack.

Case V.—*To measure a stack with rectangular base, prismoidal body, and sloped ends.*

The cubic content of the body is found as in Case IV. To find the content of the top with the sloped ends, take twice



the length at the eaves, and add the length at the top; multiply this sum by the breadth at the eaves, and again by the perpendicular height from the eaves to the top, and one-sixth of the product will be the content of the top.

Measurement of Timber.

The Table of Timber or Wood Measure is set out on page 15.

Timber less than 26 inches in circumference (or girt) is generally sold at an inferior price to timber exceeding that girt. The girt is taken with a piece of whip-cord, which, when divided into four, gives the quarter-girt, the dimension of which is measured with an ordinary two-foot rule. The girt must be taken in the middle of a tree, if it taper correctly; if not, several girts must be taken, from which the average girt can be ascertained.

Rule.—Take the quarter-girt in inches, deduct 1 inch for bark for every foot of girt, and square the result. This gives the mean sectional area. Multiply this by the length of the tree in feet, and divide by 144; the quotient gives the content in cubic feet.

The timber under 26 inches girt is sold as a cord of wood which is 8 feet × 4 feet × 4 feet high = 128 cubic feet.

Weights of Materials.

For convenience of calculation a few weights per cubic foot of materials are set out below :—

1 cubic foot clay = 119 lbs.
1 „ earth = 77 to 125 lbs.
1 „ sand = 100 to 117 lbs
1 „ 6 to 1 cement concrete = 130 lbs.
1 „ brickwork in lime mortar = 112 lbs.
1 „ „ cement mortar = 115 lbs.
1 „ gravel = 100 to 110 lbs.

CHAPTER XIV.

MISCELLANEOUS CALCULATIONS AND EXAMPLES.

I. *To find the area of a trapezium, when its four sides are given, two of its opposite angles being together* $= 180°$.

Let a, b, c, and d be the four sides, and $s =$ half their sum; then, $\quad A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$.

II. *To find the area of a trapezium when its two diagonals and the angles of their intersection are given.*

Let D and Δ be the two diagonals, and α the angle of their intersection; then

$$A = \frac{D\,\Delta \text{ sine } \alpha}{2}.$$

III. A B C *is a triangle, in which the base* A B *and a point* D *therein are given,* D C *is a quality line, making a given angle with* A B*; it is required to determine* C D *so that the triangle* A B C *may contain land of a given value.*

Put A D $= a$, D B $= b$, C D $= x$, $l =$ square links in an acre, sine $\angle$ D $= \sigma$, the values of the land adjacent to a and b respectively, m and n per acre, and V the given value; then the areas of the triangles A C D, B C D are respectively $\frac{a\,\sigma\,x}{2\,l}$ and $\frac{b\,\sigma\,x}{2\,l}$ and their values are $\frac{a\,\sigma\,m\,x}{2\,l}$ and $\frac{b\,\sigma\,n\,x}{2\,l}$ whence $\frac{a\,\sigma\,m\,x}{2\,l} + \frac{b\,\sigma\,n\,x}{2\,l} = V$, and

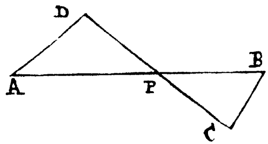
$$x = \frac{2\,V\,l}{\sigma\,(a\,m + b\,n)} = C\,D;$$

whence the position of the point C becomes known.

COR. When C D is perpendicular to A B, $\sigma = 1$, whence the above formula becomes

$$x = \frac{2\,V\,l}{a\,m + b\,n} = C\,D.$$

IV. In making an extensive survey, the fundamental lines **A** B, B C, C D, D A were measured, and the distance C P noted; but the distance A P was accidentally omitted. It is required to lay down the lines independently of this distance; and give a solution when A D = D P.



Put $A\,B = a$, $B\,C = b$, $C\,P = c$, $P\,D = d$, $D\,A = e$, and $A\,P = x$; then $P\,B = a - x$; and by trigonometry cos. $\angle A\,P\,D$

$$= \cos.\ \angle\,B\,P\,C = \frac{x^2 + d^2 - e^2}{2\,d\,x} = \frac{(a-x)^2 + c^2 - b^2}{2\,c\,(a-x)};\ \text{whence}$$

$$x^3 - \frac{a(c+2d)}{c+d}x^2 + \frac{d(a^2-b^2+c^2)+c\,(d^2-e^2)}{c+d}x - \frac{ac(d^2-e^2)}{c+d} = 0,$$

a cubic equation from which the value of x may be found, which determines the distance A P.

When $A\,D = D\,P$, that is, when $d = e$, the above equation becomes

$$x^2 - \frac{a\,(c - 2\,d)}{c + d}\,x + \frac{d\,(a^2 - b^2 + c^2)}{c + d} = 0,$$

a quadratic equation, from which the value of $x = A\,P$ may be readily found.

Investigation of formulæ.—As the formulæ for dividing land, at pages 169 170, **B** and **C**, **Case XII**., are given without investigation, the processes of deducing them are here given.

(1) By referring to the definition of the symbols, and to the diagram at page 168, **B**, it will be seen that the right-angled triangles W S s, W U u, W F b, are similar, therefore,

$$s : x :: a : \frac{a\,x}{s} = S\,s,$$

$$s : x :: a + b : \frac{(a + b)\,x}{s} = U\,u;\ \text{hence}$$

the area in acres of W S $s = \frac{1}{2}$ W S · S $s = \frac{a^2 x}{2 s l}$,

————————of S s u U $= \frac{1}{2}$ (S s + U u) S U $= \frac{(2 a + b) b x}{2 s l}$,

————————of U u b F $= \frac{1}{2}$ (U u + F b) U F $= \frac{(2 a + 2 b + c) c x}{2 s l}$

Then these areas, being multiplied by their respective values per acre, and their sum equated to the required value V′, give

$$\frac{a^2 m x}{2 s l} + \frac{(2 a + b) b n x}{2 s l} + \frac{(2 a + 2 b + c) c o x}{2 l s} = \text{V}',$$

whence $x = \dfrac{2 l s \text{V}'}{a^2 m + (2a + b) b n + (2a + 2b + c) c o} = \text{F } b.$

COR. I. If there be only two qualities of land to be thus laid out, c must be made to vanish in the above formula, whence it becomes

$$x = \frac{2 l s \text{V}'}{a^2 m + (2 a + b) b n} - \text{F } b.$$

COR. II. If there be four different qualities of land, let the breadth of the additional quality be d, and its value p; then the formula becomes

$$x = \frac{2 \quad s \text{V}'}{a^2 m + (2 a + b) b n + (2 a + 2 b + c) c o + (2 a + 2 b + 2 c + d) d p} = \text{F } b$$

the law of extension being sufficiently clear.

(2) By referring to the diagram at page 170 **C**, Case XII., and to the symbols on the same and following page, it will be readily perceived, that by drawing a perpendicular from a on N B (which is not shown in the diagram), that $a q = a - 2 x \alpha$, whence the area in acres of the trapezoid

N $a q p = \frac{1}{2}$ (N $p + a q$) $p q = \dfrac{(a - \alpha x) x}{l}$, in a similar way

the area of B $b s r$ is found $= \dfrac{(c - \beta x) x}{l}$, the area of the rectangle $p q s r$ being obviously $= \dfrac{b x}{l}$. By multiplying these

three areas by their respective values, and putting their sum equal to the required value V′, their results

$$\frac{(a - \alpha x)\, m x + (c - \beta x)\, o x + b n x}{l} = V' \text{; whence}$$

$(\alpha m + \beta o)\, x^2 - (a m + b n + c o)\, x + l V' = 0$; and, by solving this quadratic for the reciprocal of x, there results

$$x = \frac{2 l V'}{am + bn + co + \sqrt{(a m + b n + c o)^2 - 4 (\alpha m + \beta o)\, l V'}} = p q \text{ or } r s;$$

NOTE.—The method of adapting this formula to a greater or less number of qualities of land is sufficiently clear, from the different modifications of the preceding formula in Cor. I. and II.

V. The side of an equilateral triangle is $= a$, and its area $= A$; prove that

$$A = \tfrac{1}{4} a^2 \sqrt{3}.$$

VI. The base of an isosceles triangle is $= a$, one of its equal sides $= b$, and its area A; prove that

$$A = \tfrac{1}{4}\, a \sqrt{(2 b + a)\,(2 b - a)}.$$

VII. In a triangle are given the perpendicular $= p$, the angles opposite the perpendicular $= \alpha$, and β, and consequently the third angle $= \gamma$; prove that

$$A = \frac{p^2 \text{ sine } \gamma}{2 \text{ sine } \alpha \text{ sine } \beta}.$$

VIII. A B C D is a trapezium, in which the angles at A and C are equal, and $AB = a$, $BC = b$, $CD = c$, $DA = d$, the half sum of all the sides $= s$, and the area $= A$; then prove that

$$A = \frac{a d + b c}{a d - b c} \sqrt{s (s - a - c)\,(s - b - c)\,(s - c - d)}.$$

IX. n a trapezium A B C D, are given two opposite sides, $AB = a$, $CD = b$, the angles at A, B, and C, respectively $= \alpha$, β, and γ, and consequently the fourth angle D $= \delta$; then prove that

$$A = \frac{a^2 \text{ sine } \alpha \text{ sine } \beta - b^2 \text{ sine } \gamma \text{ sine } \delta}{2 \text{ sine } (\alpha + \beta)}$$

X. In a trapezium A B C D, are given the three sides C B = a, B A = b, A D = c, and the angles at A and B respectively = α and β; then prove that

$$A = \frac{1}{2}\left\{ a\,b \text{ sine } \beta + b\,c \text{ sine } \alpha - a\,c \text{ sine } \alpha + \beta) \right\}$$

XI. In a pleasure ground A B C D, known to be rectangular, only the following dimensions could be taken, on account of obstructions from buildings, shrubberies, and ponds, viz., the distance A E to the point where the perpendicular B E falls on the diagonal A C, and the prolongation E F of B E till it meet the side C D; it is required to find the area of the pleasure ground, when A E = 32, and E F = 4 chains. *Ans.* 64 acres.

XII. A large building is known to be of a square form, but no one of its sides could be measured on account of obstructions from other buildings; however, from a point P three streets diverge directly to its three nearest angles A, B, and C. Now P A = 60, P B = 40, and P C = 70 yards; required the side of the building.

XIII. Within a rectangular garden, the length of which is 4 and its breadth 3 chains, there is a piece of water in the form of a trapezium, the opposite angles of which are in a direct line with those of the garden, the distances of these opposite angles, taken in succession, are 20, 25, 40, and 45 yards; required the area of the water.

Ans. 960 square yards.

XIV. Given the three perpendiculars of a triangle, from the angular points to their opposite sides, 10, 11, and 12 chains, to find the area of the triangle.

Ans. 7*a.* 0*r.* 4½*p.*

XV. Three objects, A, B, C, are observed at a point D, exterior to them; the distances of the objects are known to be as follows, A B = 8, B C = 12, and A C = 7⅙ miles: the angles A D C, A B D are respectively 25° and 19°; required the distances A D, B D, and C D.

XVI. A gentleman intends to make an elliptical garden

the principal axes of which are to be as 3 to 1, and the fence of which is to include three trees, one at the end or the transverse axis, the second 6 poles from it, and the third the same distance from the second, the three trees forming a right angle at the second; required the axes and area of the garden.

Ans. Major axis 16·7613 *poles, minor axis* 5·5871 *poles, area* 73·55 *sq. poles.*

XVII. In an elliptical enclosure of one acre the principal axes are as 5 to 4; required the length of a chord, which, fastened to the end of the longer axis, will allow a horse to graze half an acre.

Ans. 48·45473 yards.

XVIII. Given the diagonals and two opposite sides of a trapezium to construct it, when the area is a maximum or a minimum.

XIX In a trapezium A B D C are given all the sides, A B $= a$, B D $= b$, D C $= c$, C A $= d$, and the diagonal B C $= e$, to find the diagonal A D, without constructing the figure.

$$\textit{Ans.}\ \mathrm{A\,D} = \sqrt{a^2 + b^2 - a\,b \times \frac{g\,h - \sqrt{4 - h^2} \times \sqrt{4 - g^2}}{2}}$$

$$\text{in which } g = \frac{a^2 + e^2 - d^2}{a\,e}, \text{ and } h = \frac{b^2 + e^2 - c^2}{b\,e}$$

XX. There are given the four sides of a trapezium, and a line joining given points in two of its opposite sides, from which it is required to construct the figure.

XXI. The five sides a, b, c, d, and e of an irregular field are given, in which the angles between a and b, b and c, c and d, are equal but not given; from these *data* it is required to lay down the field.

XXII. A gentleman has an elliptical garden, the principal axes of which are 50 and 40 yards, enclosed by a brick wall 13 feet high. He ordered his gardener to place his seat at equal distances from the centre, one of the foci and the boundary of the garden and around the seat to make a gravel walk of equal breadth taking up $\frac{1}{20}$ of the

area of the garden, and to be of such a nature, that, while the gentleman is seated, and the gardener moving along the middle of the walk, the gentleman's eye, the gardener's utmost height, and the top of the wall, may be in the same straight line; the height of the gentleman's eye (when seated), and that of the gardener, being 4 and 6 feet respectively. Required the position of the seat, and the nature and breadth of the walk.

Ans. The equal distances of the seat from the centre, focus, and curve = 10·7812 yards; the walk is elliptical, its axes being 11⅓ and 9⅓ yards, its breadth 2½ yards, and the position of the walk with respect to the seat being the same as the fence of the garden.

XXIII. In a triangle A B C, A B $= a$, B C $= b$, C A $= c$, and from D, a given point in A B, the distance D B $= d$, D E a line meeting B C and dividing the triangle, so that Δ A B C : Δ B D E :: m : n; then prove that

$$\mathrm{BE} = \frac{a\,b\,n}{d\,m}$$

But if it be found from the calculation that B E is greater than B C, and that the divisional line will meet A C in some point F; then if the triangle A B C : the figure B D F C :: m : n, prove that

$$\mathrm{A\,F} = \frac{a\,c\,(m-n)}{m\,(a-d)}.$$

XXIV. A B C D is a given trapezium, and T a given point in A D, from which a line T E is drawn to meet B C in E, so that trapezium A B C D : trapezium D C E T :: m : n. Now, since the trapezium A B C D is given, if the sides A D, B C be prolonged till they meet in Z, the point Z will also be given; therefore, put A Z $= a$, B Z $= b$, D Z $= c$, C Z $= d$, and T Z $= f$; then prove that

$$\mathrm{E\,Z} = \frac{n\,(a\,b - c\,d)}{m\,f} + \frac{c\,d}{f}$$

XXV. It is required to divide a given trapezium into four equal parts, by two straight lines perpendicular to one another.

XXVI. The widths of a laterally sloping railway cutting from the centre of the line are expressed generally by the following formula:

$$w = \frac{b\,s}{\pm\,r\,h}$$

the positive sign being used for the width measure down the slope, and the negative one for that up the slope, in which formula $b = \frac{1}{2}$ width of the cutting, assuming its surface to be level, h = difference of level readings at the distances s and l on the slope and level, and r the ratio of the slopes. See figure page 147.

Let a and b be the depth of a railway cutting to the intersection of the slopes, l = length of the cutting, $w = \frac{1}{2}$ bottom width, all in feet, and r = ratio of the slopes, the surface of the cutting being assumed to be horizontal, then prove that the content of the cutting in cubic yards is

$$\frac{l\,r}{81}\left(a^2 + a\,b + b^2 - \frac{3\,\omega^2}{r^2}\right)$$

XXVII. When a and b are the depths of a horizontal cutting to the formation level, and the other dimensions the same as in the last Example, then prove that the content of the cutting in cubic yards is

$$\frac{l\,r}{81}\left\{a^2 + a\,b + b^2 + \frac{3\,\omega}{r}\,(a + b)\right\}$$

XXVIII. Let A and B be the areas of the cross sections of a cutting to the intersection of the slopes, d the area of the end of the prism below the formation level, and l the length of the cutting, then prove that the content in cubit yards is

$$\frac{l}{81}\,(\mathrm{A} + \mathrm{B} + \sqrt{\mathrm{A\,B}} - 3\,d)$$

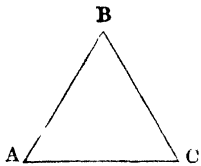
XXIX. Let the dimensions be as in Example XXVI; then prove that the error in defect of the method of finding the content of a cutting by using the mean depths is

$$\frac{l\,r}{324}\,(a - b)$$

XXX. Let the dimensions be as in Example XXVIII; then prove that the error in excess of the method of finding the content of a cutting by using mean areas is

$$\frac{l}{162}(\sqrt{A} - \sqrt{B})^2$$

When plotting a triangulation survey a point of origin is chosen right out of the survey, to the S.W., so that every point in the survey is situated to the N. and E. of it. The position of each station is then worked out into rectangular co-ordinates and stated thus :—



	N.	E.		
A =	4967′	2835′	Given ∠A =	48° 12′
			„ ∠C =	62° 47′
C =	4831′	5382′	„ ∠B =	69° 1′
				180° 00′

The position of the point B is required. From the given co-ordinates C is 136′ S. of A and 2547′ E. of A.

Then log 2547 =	3·4060289		3·4060289
log 136 =	2·1335389	Log sin 86° 56′ 36″ =	9·9993817
	1·2724900		3·4066472
Log tan 86·56 =	11·2710411	Whence AC =	2550·63 ft.
	14489		
	60		
Diff. 23712)869340(36″			

Bearing AC = S.E. 86° 56′ 36″, ∴ AB = N.E. 44° 51′ 24″; BC = N.W. 24° 9′ 36″.

From above log AC =	3·4066472	Again =	3·4066472
Log sin 62° 47′ =	9·9490402	Log sin 48° 12′ =	9·8724337
	13·3556874		13·2790809
Log sin 69° 1′ =	9·9702002	Log sin 69° 1′ =	9·9702002
∴ Log AB =	3·3854872	Log BC =	3·3088807
Log cos 44° 51′ 24″ =	9·8505687	Log cos 24° 9′ 36″ =	9·9601882
Log N. 1722·09 =	13·2360559	Log N. 1858·1 =	13·2690689
	3·3854872		3·3088807
Log sin 44° 51′ 24″ =	9·8483958	Log sin 24° 9′ 36″ =	9·6120270
Log E. 1713·49 =	13·2338830	Log W. 833·50 =	12·9209077

	N.	E.		N.	E.
A =	4967	2835	C =	4831	5382
	1722·09	1713·49		1858·1	833·50
B =	6689·09	4548·49	B =	6689·1	4548·50

The position of B is thus fixed from two points, and the calculations check each other.

CHAPTER XV.

PUBLICATIONS OF THE ORDNANCE SURVEY.

Historical Sketch.—The Ordnance Survey Department of Great Britain and Ireland publish for the use of the public a series of maps of the entire Kingdom, drawn to various scales, as set out in tabular form on the next page. These are known as Ordnance Maps. The name Ordnance Survey was derived from the fact that, up till 1855, the survey was conducted by the Honourable Board of Ordnance, which is now abolished. The survey is now under the control of a Director-General, responsible to the Board of Agriculture.

The survey appears to have originated for military purposes in 1747, after the crushing of the 1745 Rebellion, and was first commenced in the Highlands of Scotland, the heart of the insurrectionary movement. It was ultimately extended to the whole of Scotland, excluding the Islands.

Owing to the inferiority of the instruments used, the results first obtained presented in reality more a magnificent military sketch than an accurate plan.*

In 1787 a base line was measured on Hounslow Heath, near London, and the triangulation of France connected with that commenced in Great Britain. Another base line, 5½ miles long, was also measured the same year on Romney Marsh, in Kent, for a base of verification; and in the summer of 1794 another base of verification, 7 miles long, on Salisbury Plain, was also measured. On the last-mentioned base, the main triangulation of the United Kingdom rests. The triangulation of the accurate Survey of Scotland was commenced in 1809, and that of Ireland in 1825.

The Ordnance Survey of the Kingdom is now complete

* See "The Ordnance Survey of the United Kingdom," by Colonel T. P. White, R.E.

and the maps are revised and brought up to date at intervals.

The extreme accuracy of the survey may be judged from the fact that, when the base of Salisbury Plain was computed from the Irish base, 350 miles distant, through a long intervening chain of triangles, the computed base differed from the measured length by less than 5 inches.

The Ordnance Maps of Great Britain are published at Southampton, and those of Ireland in Dublin.*

The following is a list of the principal Maps published :—

Natural Scales.	Inches to One Statute Mile.	Statute Miles to One Inch.	Chains to One Inch.	Remarks.
$\frac{1}{500}$	126·720	0·0079	$0{\cdot}6\dot{3}\dot{1}$	Town Maps.
$\frac{1}{528}$	120·	$0{\cdot}008\dot{3}$	$0{\cdot}\dot{6}$	
$\frac{1}{1056}$	60·	$0{\cdot}01\dot{6}$	$1{\cdot}\dot{3}$	
$\frac{1}{2500}$	25·344	0·0395	$3{\cdot}1\dot{5}\dot{6}$	Cadastral or Parish Maps of the United Kingdom.
$\frac{1}{10560}$	6·	$0{\cdot}1\dot{6}$	$13{\cdot}\dot{3}$	County Maps of the United Kingdom.
$\frac{1}{63360}$	1·	1·	80·	General Map of the United Kingdom.

The **General Map,** on the scale of 1 inch to a statute mile, is useful for military purposes, or for reading the character of a large district. In it are shown the roads, railways, towns, villages, &c. It is published as follows :—

(1) For England and Wales in two sets of maps—

(*a*) The maps of the original survey on a scale of 1 inch to a mile. The maps are uncoloured, and the hills engraved.

(*b*) The new 1-inch General maps, reduced from the 6-inch maps. These maps can be had either coloured or uncoloured, and with or without hill shading. The contour lines, or lines of equal elevation, are marked.

* The London agent is Mr. E. Stanford, Long Acre, London, W.C.

(2) For Scotland, the maps of the original survey. They have the contour lines marked, and can be had coloured or uncoloured, and with or without hill shading.

(3) For Ireland. These maps are reduced from the 6-inch scale, and can be had coloured or uncoloured. Generally speaking, they are issued without contours.

The **County Maps,** on the scale of 6 inches to 1 statute mile, are of great assistance for estate or engineering purposes, the information on the 1-inch maps being amplified. The maps for the United Kingdom are published uncoloured, and, with rare exceptions, without hill shading. The contours are marked, and also the position and relative altitude of the Ordnance bench marks.

The **Parish Maps,** on the scale of 25·344 inches to the statute mile, or $\frac{1}{2500}$ (= 208·33 feet to 1 inch), attain to an almost inconceivable ideal. One square inch on the paper represents as nearly as possible one acre on the ground. All the matter of the 6-inch maps is reproduced, with the exception of the contour lines, whi h is rendered unnecessary by levels indicated on the highways shown on the maps at frequent intervals. Every parcel of land is numbered, and its area computed.

In maps published since 1884 the area is given beneath the number on the map. In maps published previous to that date the area is given in area books published with the maps. The maps can be had either coloured or uncoloured.

Maps of the whole of England and Wales are published on this scale, with the exception of moorlands and uncultivated portions. Similarly, parish maps of the greater part of Scotland, and of some parts of Ireland, are also published.

Parish Indexes are published with these maps, and give very useful information. They show the position practically of every farm in a parish, and give also the boundaries and names of every parish represented on the index.

The **Town Maps.**—As will be seen from the Table on last page, these maps (for the United Kingdom) are drawn

to three scales—viz., 126·720 inches to 1 statute mile, or $\frac{1}{500}$; 120 inches to 1 statute mile or $\frac{1}{528}$; and 5 feet to the mile, or $\frac{1}{1056}$. No new surveys of maps are now plotted to the scale of $\frac{1}{528}$, as the $\frac{1}{500}$ is more convenient, and maps of few towns only can be had on this scale (viz. $\frac{1}{528}$).

Maps of some towns can be had on both scales of 5 feet and 126·720 inches to the mile, respectively, but the latter has in recent years been substituted for the former 5-feet map. The maps can be had coloured or uncoloured.

The importance of the $\frac{1}{500}$ map can be seen from the fact that frequent levels and bench marks are given, and that it represents a plan on the scale of 41·6 feet to an inch, which is ample for a building plan, so that drains, pipes, and other details can be marked thereon with the greatest nicety.

Town Indexes are published with these maps, showing the sheets into which each town is divided.

Engraving, &c.—The 1-inch maps are engraved on copper, the remainder are produced by photo-zincography, the reduced photographic negative of the plan being transferred by a carbon print to a zinc plate from which the map is to be printed.

Official Catalogue. — The Director-General of the Ordnance Survey publishes a Catalogue* of the Ordnance Maps with the price of each and their various scales.

Levels on the Ordnance Maps.—The levels on the Ordnance Maps are referred to a datum known as Ordnance datum, which is explained in the Chapter on Levelling in this work, page 117.

Publications of the Geological Survey.—In addition to the Maps, &c., of the Ordnance Survey mentioned above, Maps* and Memoirs of the Geological Survey are issued under the superintendence of the Director-General of the Geological Survey of the United Kingdom.

* Obtainable of Mr. E. Stanford, Long Acre, London, W.C.

APPENDIX.

EXAMINATION PAPERS.

By permission of the Secretary and Council of the Institution of Surveyors, the Royal Agricultural Society of England, and the Incorporated Association of Municipal and County Engineers respectively, the following papers, set at recent examinations of those authorities, are reproduced as a guide to students.

I. EXAMINATION PAPERS OF THE INSTITUTION OF SURVEYORS, 1897.

LEVELLING (FIELD WORK).

Directions.

Take section over course marked out. Chain a line set out over rough ground, giving intermediate distances. Take angles with the theodolite.

SURVEYING AND LEVELLING.

Time allowed—Three hours.

NOTE.—*All Candidates are required to attempt questions Nos.* 1, 2 *and* 3. *Candidates other than Building Candidates will receive full marks for any* 10 *questions correctly answered. Building Candidates will receive full marks for any* 8 *questions correctly answered. Candidates omitting to leave figures by which results are arrived at, will risk a loss of marks in case of a wrong answer being given through accident. Questions* 1, 2, 3, 6, 7 *and* 9 *carry higher marks than the remainder.*

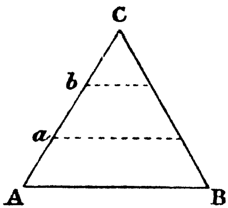
1. On the plan given, draw in pencil the lines it would be necessary to run, to enable you to make a complete survey with the chain only.

2. Compute the areas of the enclosures in the corner of the plan above mentioned, giving the results in acres, roods, and perches; one of these enclosures must be computed by means of the ordinary plotting scale, and the other in any

way the Candidate may elect. (Enclosure No. 1, if well done and a correct answer arrived at by the ordinary plotting scale, will carry full marks).

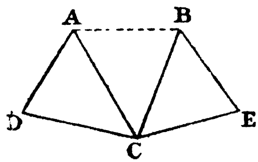
3. From the field notes given, lay down the survey lines and plot a plan to a scale of 2 chains to an inch.

4. Required to set out a circular space for a Reservoir to contain 1 acre, 1 rood, and 20 perches ; give the radius in links.



5. Divide the triangle A B C into three equal portions by lines parallel to the side A B. A B = 2,500 links ; A C = 2,100 ; and B C = 1,800. Give the area of A B C, and the distances A *a*, *a b*, and *b c*.

6. The points A and B are only both visible from one point C. Lines C D = 1,260 links and C E = 1,040 links were run, and the following angles were taken, viz. :—
A D C = 67° 30′, A C D = 45° 0′, A C B = 70° 20′, B C E = 39° 10′ and B E C = 81° 50′. Find the length A B in links



7. Plot the above figure to a scale of 1 chain to an inch, and give the distance A B as it measures upon your plan.

8. A traverse round a wood is as follows :—

A to B =	290 links,	bearing	255° 5′	A . . B
B to C =	1,000 „	„	194° 10′	
C to D =	680 „	„	77° 12′	D . . C

give the calculated distance D to A.

9. Protract and plot the above to a scale of 1 chain to an inch.

10. Convert 17 acres, 1 rood, and 20 perches (statute measure) into square yards.

11. How would you determine the latitude of any position (on land), and what instrument would you require?

12. Illustrate and describe in what way you would produce a survey line obstructed by a large tree or building.

13. If a plan is plotted to a scale of 3 chains to an inch, what proportion does the area of the plan bear to the ground ?

AFTERNOON PAPER. *Time allowed—Two hours and a half.*

NOTE.—*All Candidates are required to attempt questions Nos.* 1 *and* 2. *Candidates other than Building Candidates will receive full marks for any* 9 *questions correctly answered. Building Candidates will receive full marks for any* 7 *questions correctly answered. Candidates omitting to leave figures by which results are arrived at will risk a loss of marks in case of a wrong answer being given through accident. Questions* 1, 2, 8, 9, *and* 11 *carry higher marks than the others.*

1. Make up the level book on the back of this sheet.

2. Plot the following section to a horizontal scale of 2 chains to an inch, and to a vertical scale of 20 feet to an inch :—

Distances. Chains.	Height above Base. Feet.
0·00	30·00
1·00	31·30
2·00	33·40
3·50	27·85
5·00	27·01
6·20	30·47
7·00	23·43
9·00	26·30
10·00	23·20
11·20	25·20
12·00	29·30
14·00	31·25
15·00	33·31
15·60	34·16
17·50	32·27
18·00	30·25
18·50	29·09

3. In setting out the centre line for a new road or a railway, illustrate and describe in what way you would proceed to connect two pieces of straight by a curve of say, 10 chains radius.

4. Before commencing to take a series of levels briefly describe how you would ascertain if your level was in adjustment.

5. The point A being inaccessible and at a considerable altitude above the surrounding country, illustrate and describe in what way you would ascertain its height above the point B (the nearest convenient point of observation), using a theodolite for the purpose.

6. Give the rates of inclination between the given points of level taken upon a line chained along the invert of a watercourse.

Distance. Chains.	Height. Feet.
0·00	41·00
2 40	42·16
3·00	42·50
6·30	44·20
8·00	45·70

7. What is the rate per chain (in feet and decimals) of a gradient rising 1 in 250?

8. Give the levels of points B, C, and D on a continuous section, the level of point A being 25 feet, and the horizontal distances and angles as follows:—

A to B, 12 chains; angle of elevation, 3° 20′
B to C, 9 ,, ,, depression, 4° 25′
C to D, 15 ,, ,, elevation, 2° 15′

9. The telescope of a theodolite set 4·25 feet above the point A having a level value of 25 feet, is directed towards the bottom of a staff at B, and shows an angle of elevation of 10° 4′; it is then directed to 10 feet on the staff, when it shows an angle of elevation of 10° 35′. Required the horizontal distance A to B in feet, and also the level of point B.

10. Illustrate by diagram the difference between "true" and "apparent" level, and give a rule for determining same.

11. Construct a triangle A B C, having its sides A B = 3 inches, B C = 2½ inches, and A C = 1½ inches. Suppose the points A, B, and C to be trigonometrical stations of a survey, and that from a point D of a traverse A bears 120°, B 150°, and C 165°, find the point D by construction.

12. Explain and illustrate by diagram how you would obtain the distance to an inaccessible point, using only chain and poles.

LEVEL BOOK FOR QUESTION NO. 1.

Back Sight.	Intermediate	Fore Sight.	Rise.	Fall	Reduced Levels.	Distance.	Remarks.
					Feet.	Chains.	
6·60					45·80	0	
	4·00					1·00	
	5·70					2·00	
·80		12·20				3·00	
	6·90					4·00	
	11·20					5·00	
·24		13·12				6 00	
	4·80					7·00	
	8·30					8·00	
1·10		13·75				9·00	
	6·70					10·00	
	5·70					11·00	
	8·10					12·00	
2·90		15·05				13·00	1st side of
	7·10					14·00	Pond water
	10·60					14·30	level
	11·70					15·00	
	10·80					16·00	2nd side of
	7·10					16·40	Pond water
13·75		6·85				17·00	level.
	11·10					18·00	
	8 60					19·00	
	2·30					20·00	
		·85				21·00	

NOTE.—*A candidate before presenting himself for examination is required to do certain preliminary work in Surveying and Levelling, particulars of which can be obtained from the Secretary of the Institution.*

MENSURATION.

Time allowed—Two hours.

1. How many rods of brickwork are there in a circular pier 4 feet in diameter and 20 feet in height?

2. A circular water-tank is 12 feet internal diameter, and is 10 feet deep. A drawing of it was made to a scale of $\frac{1}{8}$-inch to a foot. Someone carelessly scaled it with a

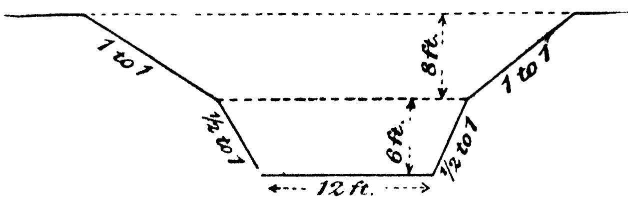
scale of $\frac{3}{8}$-inch to a foot. What error would be made in calculating the number of gallons contained in the tank when full?

3. A road rises with a gradient of 1 in 75 from its commencement to a point distant $1\frac{1}{2}$ miles (on a horizontal datum). It then falls with a gradient of 1 in 100 to its termination at a further distance of 140 chains (on a horizontal datum). What is the difference of level between the beginning and the end of the road?

4. The air in a room 30 feet × 25 feet × 10 feet has to be changed three times in an hour by air conveyed through a pipe 6 inches in diameter. At what velocity must the air move in the pipe to do this?

5. A shower of rain is registered to give $1\frac{1}{8}$ inches. How many gallons would have fallen on a field containing 100 acres?

6. What is the sectional area of a cutting with slopes as shown in the sketch, and how many cube yards are there in a chain of this cutting?



7. A railway bank is half a mile in length, and is 20 feet above the ground at one end, and 30 feet above the ground at the other. The slopes are 2 to 1 throughout. How many acres of ground does it cover?

II. EXAMINATION PAPER

OF THE

ROYAL AGRICULTURAL SOCIETY OF ENGLAND.

MENSURATION AND LAND SURVEYING.

MAXIMUM NUMBER OF MARKS, 200. PASS NUMBER, 100.

Time allowed—Three hours.

N.B.—*The answers are to be written upon one side only of the sheets supplied. The Candidate is required to write his number upon each sheet at the right-hand top corner; to see that the sheets are paged consecutively at the top-centre of each sheet; and to fasten the sheets together at the left-hand top corner, care being taken not to cover up the numbers of the questions.*

The name of the Candidate is not to be written upon any of the sheets.

Number of Candidate ____________

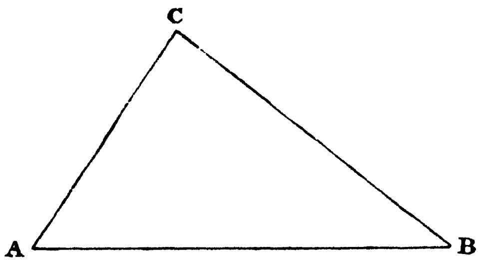
1. On the plan given on page 3 of this paper, draw in pencil the chain lines you would run to enable you to make and plot a complete survey without the aid of angular instruments.

NOTE.—*The Candidate must not spend more than twenty minutes over this question.*

2. Compute the area of the enclosure marked I on the plan given on page 3 of this paper, using the ordinary plotting scale, and giving the result in acres, roods, and perches.

3. Make up the level book on page 2, filling in the rises, falls, and reduced levels (heights above base).

4. In the triangle A B C, the length of the side A B is 1,000 links, the angle A B C = 40° 30′, the angle C A B = 70° 25′. Give the length of the sides A C and C B.



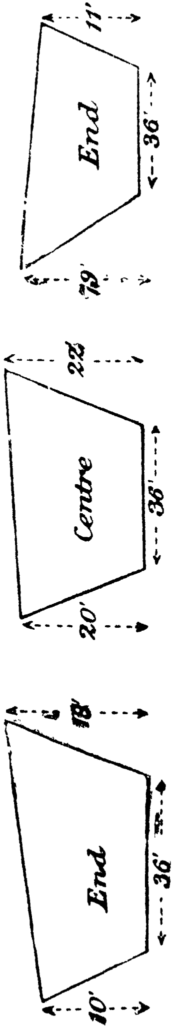
5. Calculate the number of cubic yards of earthwork in a railway cutting 4 chains in length, the ends and centre being of the sections shown below [see opposite page], and side slopes $1\frac{1}{2}$ to 1.

6. Describe the pocket sextant, and the method of adjusting it.

7. What are the scales used in the published maps of the Ordnance Survey of England ?

8. What course would be followed by the line of sight of a theodolite placed at Greenwich, and having its telescope directed at right angles to the meridian, the line of sight being indefinitely produced ?

What course would be followed by a line laid out from the same starting point and in the same original direction, and continued indefinitely from station to station by similar compass bearings ?


11'
End
36'
19'
22'
Centre
36'
20'
18'
End
36'
10'

III. EXAMINATIONS OF THE INCORPORATED ASSOCIATION OF MUNICIPAL AND COUNTY ENGINEERS.

LAND SURVEYING AND LEVELLING.

1. Show chain lines necessary for surveying plot below; also give form of field book showing the imaginary readings when making the survey.
(Sketch of an irregular polygon was given.)

2. Explain and give sketch how to ascertain the width of a river by using the chain only upon one side of the river.

3. Give at least twenty levels along a line having four changes of the instrument, and enter the imaginary levels in a sketch page of a level book.

APPENDIX IV.

Transit Theodolite Adjustments.—The purpose of this note is to describe as simply as possible, but at the same time completely, a practical way in which the adjustments under consideration can be tested and made. It may be mentioned that various methods are advocated from time to time, but, generally speaking, they are singularly inadequate and unsatisfactory from the point of view of the serious student. In making the selection of methods which follows, the writer has been guided chiefly by his own practical experience and by the following considerations:

(*a*) Convenience, or, in other words, such that can be used under almost any circumstances;

(*b*) Simplicity, accuracy, and completeness.

The adjustments are dealt with in the order that is best adopted when actually performing the operations. The instrument having been set up firmly on solid ground, the adjustment of the bubble on the horizontal plate is examined first. Nearly all modern instruments are of the three-screw pattern. In such a case, place the level parallel to two of these screws and bring the bubble to the centre. It will generally be most convenient to next turn the instrument through 90° so that the bubble is parallel to the line through the remaining screw and the centre of the instrument. Bring the bubble to centre with this screw only, and then turn the instrument back into its original position. It may be necessary to slightly adjust the bubble again to centre. Having done this turn the instrument through an angle of 180°. The bubble should again settle in the centre of its run, but if it does not, and being satisfied that this is not through inaccurate levelling on the operator's part, with the foot screws, the level should be adjusted by the lock nuts which secure it in position. When making this adjustment, bring the bubble halfway back to centre

by the foot screws, and then centre it exactly with the lock nuts. The operation may require two or three repetitions, but when properly performed the bubble will remain in the centre of its run with the telescope pointing in any direction, and with the lock nuts tightened up. If there is a second small bubble, either on the plate or near the base of the standards which carry the telescope, this should now be in the centre of its run. If it is not, make it so by the lock nuts only, and adjust it until this also remains in the centre through a complete revolution of the instrument.

The main bubble may now be examined. If this is carried on the vernier arm to the vertical circle, it should now be in the centre of its run. If it is not, revolve the instrument gently through a complete circle, and see if the bubble remains in the same position in its tube. If it does, it shows that although the bubble is out of adjustment, the vertical axis of the instrument is truly vertical. In any case, bring the bubble to centre by means of the clipping screws H, H′ in the figures on pages 61 and 63. Turn the instrument through 180° and the bubble will remain centred, when the axis is truly vertical. If the bubble moves slightly, bring it halfway back to the centre with the levelling screws and the other half by the clipping screws. Repeat until this bubble remains centred through a complete revolution. The smaller bubbles previously considered may now be compared with the more sensitive bubble, and in the event of their requiring any slight re-adjustment, it should be made with the lock nuts only.

When there is no bubble on the vernier arm, the main level is carried on the telescope. If this is the case, set the verniers to read zero on the vertical circle, with minute accuracy. Proceed then exactly as if it were on the vernier arm. Many theodolites have a similar bubble on both vernier arm and on the telescope. In such, the bubble on the vernier arm having been set as described previously, that on the telescope should also come to the centre when the verniers to the vertical circle are set to zero. If it does not, leave it for the moment. So far we have no knowledge as to whether the bubble on the vernier arm or that on the telescope is incorrect. The next test is that of collimation. The line of collimation is that line of sight which passes exactly through the optical axis of the instrument. The

lines of sight given by the stadia lines in a telescope are not lines of collimation, and if the intersection of the webs in a transit theodolite is not situated exactly in the optical axis, it is not in the line of collimation. The mechanical axis of the telescope should be coincident with the optical axis. If it is not, so long as the two axes are parallel it does not matter to us, as they will be so near each other when the instrument is by a good maker that we cannot detect any error caused thereby. In a transit theodolite the line of collimation must be considered, both in azimuth and in altitude.

Take the former first. To test this, select any clearly defined object on to which the webs can be accurately sighted. Clamp the instrument on to this object, both with the axis clamp and that to the vernier plate. Transit the telescope, i.e. unclamp the vertical circle and revolve the telescope in its bearings until it points in the opposite direction. An assistant now places a mark, such as a ranging rod, at a considerable distance, or if more convenient, a fine pencil line on paper pinned to a drawing board, if nearer at hand. This mark must be vertical, and the assistant is directed until his mark is exactly cut by the webs. If the collimation in azimuth is correct, the distant mark, the centre of the instrument, and the assistant's mark will all lie in one straight line. To test if this is so, unclamp the main axis, turn the instrument through 180°, sight on to the back station again, and re-clamp the instrument. Transit the telescope as before, and sight towards the assistant's mark. If the webs are properly adjusted, their point of intersection will again come on to that mark, although the telescope is inverted to what it was before. If it does not, let the assistant make another mark in the new line of sight, without disturbing that previously made. The assistant than measures the distance between his marks and sets a third at exactly a quarter of that distance from the last mark made. The diaphragm is then moved by the horizontal pair of collimating screws until its point of intersection is on the third mark. A few repetitions of the above may be necessary before the adjustment is made perfect.

To test the collimation adjustment in altitude any convenient back station may again be taken, but it should be

such that a mark can be made in front which can be observed without disturbing the telescope in altitude. As an instance assume the ground is level. If no natural object is available, have a ranging rod fixed for the back station. The instrument being properly levelled, the telescope is directed to the back mark and another mark fixed in front by transitting the telescope as before. Next, turn the telescope on to the back station and adjust the horizontal web, by means of the clamp and tangent to the vertical circle, exactly on to, say, one of the colour divisions of the staff or some other easily identified mark. Unclamp the lower axis and revolve the instrument through 180° until the webs cut the vertical mark fixed in front. A levelling staff can now be held alongside this mark, if at a distance, or a drawing office scale fixed, if near. The reading of the horizontal web is then taken. The telescope is now transitted and adjusted with the horizontal web again on the mark at the back station. Unclamp the axis and again turn the instrument through 180°, clamping it with the intersection on the vertical mark. The horizontal web should now give the same reading on the staff or scale as before. If it does not, take the difference of the two readings, and set the webs to read quarter of this difference from the last reading taken, using the vertical pair of collimating screws for the purpose. As it may be necessary to slacken off one of the horizontal collimating screws to enable this to be done, the value of having the vertical reference mark will be seen, as it can be observed, after all the collimating screws have been tightened up, whether the adjustment in azimuth is still correct. Previous remarks as to repetition apply here also.

The foregoing adjustments having been skilfully made, the intersection of the webs will come on to the same marks, both in azimuth and in altitude, whether the telescope is in its normal position or inverted. The above methods have been chosen as they are applicable either to the ordinary telescope or to the internal focussing variety.

When the webs are being adjusted, care should be taken to see that the horizontal web is horizontal. This may be tested by slowly revolving the instrument so that the web traverses some mark right across the field of view, in the telescope, and, if necessary, tapping the collimating screws

with some light object, one up and the other down, as the holes are slightly slotted for this purpose. This, of course, should be done during one of the repetitions. Care should also be taken that all the collimating screws are properly tightened up, when the adjustments to collimation have been perfected.

We now revert to the main bubble or bubbles. Select a stretch of level ground on which a distance of, say, five chains can be measured. At the extremities of this line have pegs driven in, and set the instrument exactly midway between the pegs. Level the instrument up, set the verniers on the vertical circle to read zero, and read the levelling staff when held perfectly upright, on both pegs. Assuming for the moment that we are using the main level on the vernier arm, or the one on the telescope if there is not a level on each, the difference of reading on the two pegs will indicate their true difference of level, provided the bubble is exactly centred for both back and fore sights, owing to the instrument being the same distance away from each. Say the staff reads 4 feet on one peg and 3 feet on the other, the 4 feet peg is then 1 foot lower than the other. We now take the theodolite to whichever peg the staff is at, and set it up so that the eyepiece is within half an inch of the staff after the instrument has been levelled up. Say we are at the lower of the two pegs. We now look through the object glass at the staff and a small round circle is visible through the eyepiece. The point of a pencil can be placed very easily in the centre of the small field of view and the staff reading of the pencil point taken. Say this reads 3·67 feet. Our peg is 1 foot lower than the far one, so that if the staff is removed to the latter, it should read 2·67 to be level with the instrument. If it does not, we bring the webs to this reading by the clipping screws H, H′. The line of collimation is now level, and our main bubble must be adjusted if necessary so that it is exactly centred, whether it is on the vernier arm or on the telescope. If on both, one may be found correct, but if not, both must be adjusted. The bubbles must be adjusted in this instance with their lock nuts only, as the staff reading must be maintained.

The foregoing operations having been properly performed, we should know that when our vertical axis has been set

truly vertical, all our bubbles will assume the centre position of their run when the line of collimation is horizontal, and that the vertical circle will read zero, thus eliminating any so-called index error. If the verniers to the vertical circle have been carelessly set during the bubble adjustments, an index error may exist, but it would be entirely our own fault.

Lastly, the adjustment of the transit axis may be examined. Carefully level the instrument and select some definite point at a considerable elevation that can be accurately sighted with the intersection of the webs. The point chosen need not be very high if near, but the telescope should describe a considerable vertical arc during the test. After sighting the mark, the telescope is tilted so that a mark can be fixed on the ground exactly under the elevated point so far as our line of sight is concerned. The instrument is then turned through 180° and the elevated point again sighted after the telescope has been transitted. The latter is dipped again, and if the horizontal axis is truly level the webs will come again on to the mark on the ground. If they do not, bring them half way on to it by the adjustment on one of the standards carrying the transit axis. Repeat the operation until the webs will intersect a mark on the ground in both positions of the telescope. The latter adjustment is very necessary if the theodolite is to be used for plumbing high structures such as chimney stacks, also in ordinary surveying operations when the angle is required between two points which differ greatly in elevation.

APPENDIX V.

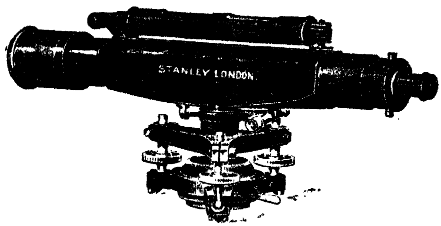
Adjustment of the Dumpy Level.—The method described in the text when testing the collimation adjustment is that usually adopted by the engineer when a level requires adjusting, excepting that the distances between the pegs should be greater. Instead of 100 feet from the centre peg, the distances should be at least 200, and preferably 300 feet. We will assume the latter figure is taken. The stretch of ground chosen must be approximately level, but it is not usual to trouble about driving the pegs absolutely level with each other. If the ground permits, this of course can be done when preferred. It is more convenient if the bubble is adjusted so that it comes to centre when the vertical axis is truly vertical and remains there during a complete revolution of the instrument. Assuming that the instrument has been set and carefully levelled over the centre peg and that the readings on the staff are at A, 3·92 feet, and at C, 4·63 feet, the peg at C will obviously be ·71 feet lower than that at A. When the instrument is taken to the position D (p. 105), we will assume that the reading on A is 4·27 feet, and that on C is ·35. The line of sight is, therefore, dipping; since C is ·71 feet lower than A the reading should be $4 \cdot 27 + \cdot 71 = 4 \cdot 98$. Subtracting the actual reading from this, viz. $4 \cdot 98 - \cdot 35 = 4 \cdot 63$, the amount of dip in our line of sight in a distance of 600 feet. The reading on A will also be lower than the true level line. The correct reading is obtained by simple proportion, thus—As $600 : 20 :: 4 \cdot 63 : x$. Whence the error is ·154 in 20 feet.

A level line from the instrument will, therefore, read 4·424 feet on the staff at A and 5·134 at C. If the level has an internal focussing telescope, the horizontal web can be brought on to the latter reading by adjusting the diaphragm to it. On completion, we have the line of sight level and the bubble in the centre of its run. The bubble will also reverse, that is, remain centred during a revolution of the telescope, according to our former premise. Such an instrument would then be in adjustment for all practical purposes.

If the telescope is one of the ordinary pattern in which the diaphragm moves relatively to the object glass during the operation of focussing, after the web has been set to read 5·134 feet at C, the staff must be brought back to A. We know that the staff should read 4·424 at this point. If it does, the instrument is in adjustment. We will assume however, that it reads 4·444. This would show that the line of sight is not in the line of collimation and that the act of withdrawing the webs from the object glass in focussing down to the near distance causes the web to move in a direction which is not parallel to the optical axis. The error we have assumed as due to this cause is ·02 feet. This is multiplied by 2 and added to the last reading, apparently increasing the error. Adding ·04 to 4·444 = 4·484. We now adjust the diaphragm by its collimating screws to read this figure. The staff is again taken to C and the diaphragm is brought to the reading 5·134 by adjusting the telescope blocks by the screws at either end of the limb at L (p. 101). This will throw the bubble out of centre and it must, therefore, be adjusted so that it comes to centre when the webs intersect the 5·134 on the distant staff. The operation having been properly performed, the instrument will read 4·424 on the near staff, but the whole operation may require repeating before the adjustment is perfect. The instrument will then have been collimated and adjusted. It may be mentioned that in the case of an old instrument, slack in the draw tubes may cause the eye end to sag when withdrawn. If this occurs, the instrument is worn out and worse than useless, unless the makers can refit the tubes. The writer begs to say that he is indebted to Messrs. Cooke, Troughton & Simms, Ltd., for the valuable method of collimating a dumpy level in the manner described. In connection with this point it may be added that in the case we assumed our error was in excess of the true reading. Had it been the other way, i.e. say the reading on the near peg had been 4·404, the error would still be ·02. In this case, however, after doubling the error, we should subtract it and set the webs to read 4·364, thus again apparently increasing the error.

We have one further type of dumpy level to consider, such as Stanley's engineer's level, illustrated herewith, in which the telescope and vertical axis is formed out of

one casting. Such an instrument could be collimated in the manner described if it required it. To bring the collimated web on to the reading of the staff at C we should have to tilt the telescope by the foot screws. The bubble could then be adjusted to centre. Such an instrument could be used to give accurate results by bringing the bubble to centre for each sight, but the bubble would not reverse, and could not be made to do so except by the maker. It would be somewhat inconvenient to use an instrument thus defective. Unless such an instrument has met with an accident at some time, it will be found to be in collimation when the web has been set to read the level line in the



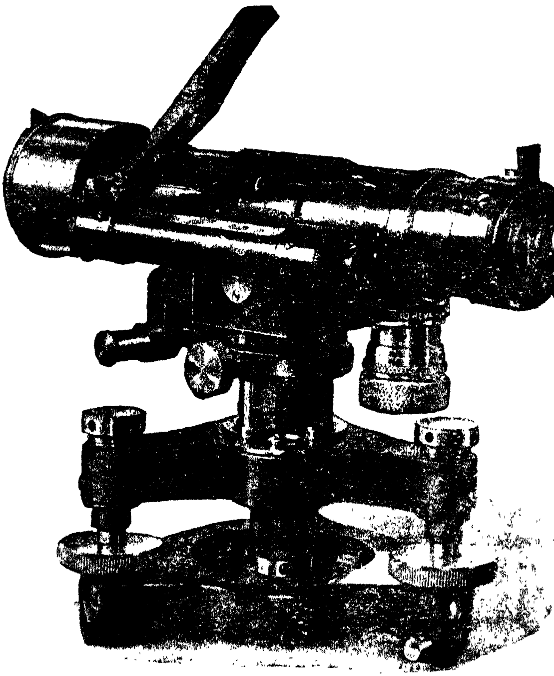
Stanley's Engineer's Level with Quick Acting Joint,

first instance, and the lack of an adjustment to the horizontal limb is not really a drawback. The instrument has to be made properly in the first instance, and the writer considers it an advantage to have removed the possibility of a somewhat awkward adjustment being tampered with.

Messrs. Cooke, Troughton, & Simms' new level is a fine example of the most modern type. In these instruments a perfectly accurate vertical axis is no longer the basis on which all results depend. In the case of the instrument illustrated, the levelling screws are rather quick in their action and a circular level is provided for use in conjunction with them. When a sight is being taken, the longitudinal level at the side of the telescope is brought exactly to centre by means of the large screw under the eye end of the telescope. The image of this bubble is reflected in an inclined mirror, and can be observed from the eye end of the telescope. This bubble is far more sensitive than most of those fixed on the older type of instrument, such refinement being rendered possible by the improved method of manipulating the instrument when in use.

APPENDIX VI.

The Plane Table.—This instrument consists essentially of a drawing board mounted on a tripod in such a manner that it can be placed in a horizontal position and clamped.

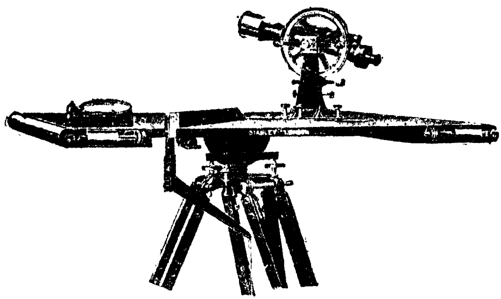


Cooke's Engineer's Level.

An alidade is used in conjunction with the plane table. This consists of a ruler provided either with sights at each end or a telescope in the better-class instruments. In any case, the line of sight passes over the fiducial edge of the ruler. The table may be set up level by means of a loose level supplied with it, such levels sometimes being attached to a compass which can be placed against the edge of the alidade to obtain the bearings of the lines.

The stations on the plan can be plumbed over the actual stations by means of a counterpoise and bent arm which supports the plumb bob underneath the table.

By the courtesy of Messrs. Stanley, Ltd., a high-class instrument of their manufacture is illustrated herewith. The method of using is very simple. The instrument is set up and levelled over the first survey station. The paper is stretched on the board and a puncture mark made with a pricker to represent the station with the aid of the plummet. The telescope is then directed to station No. 2, the line 1 to 2 being the base on which the survey is to be made. When the edge of the alidade passes through the puncture representing station No. 1 and the webs intersect



Stanley's High Class Plane Table.

station No. 2 a line is drawn on the paper from 1 towards 2. This line must be measured, or its distance known by other means. The length of this is scaled off with the appropriate scale to which the plan is required. While the board is fixed in this position lines are drawn with the alidade towards all the points which require fixing from this station, such lines all radiating from the puncture mark.

The table is then removed to station No. 2 and set up as before in such a way that after the station has been plumbed, the telescope of the alidade comes to station No. 1, when its fiducial edge is against the base line on the plan. This being so, radiating lines are again drawn from this station to the various points which require fixing, this being done by these lines intersecting the corresponding ones from station No. 1. Confusion must, of course, be

avoided, and the number or description of the station to which each line runs can be written in pencil alongside the line. With the alidade shown in the illustration distances can be measured with the stadia to points which cannot be fixed by intersection. With the plane table a good deal of detail can be picked up, but the offsetting of fence lines, etc., is performed in the usual manner from chain lines run from the various stations fixed by the plane table. The latter correspond, therefore, in a way, to the trig stations fixed by the theodolite in a triangulation.

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THE END.